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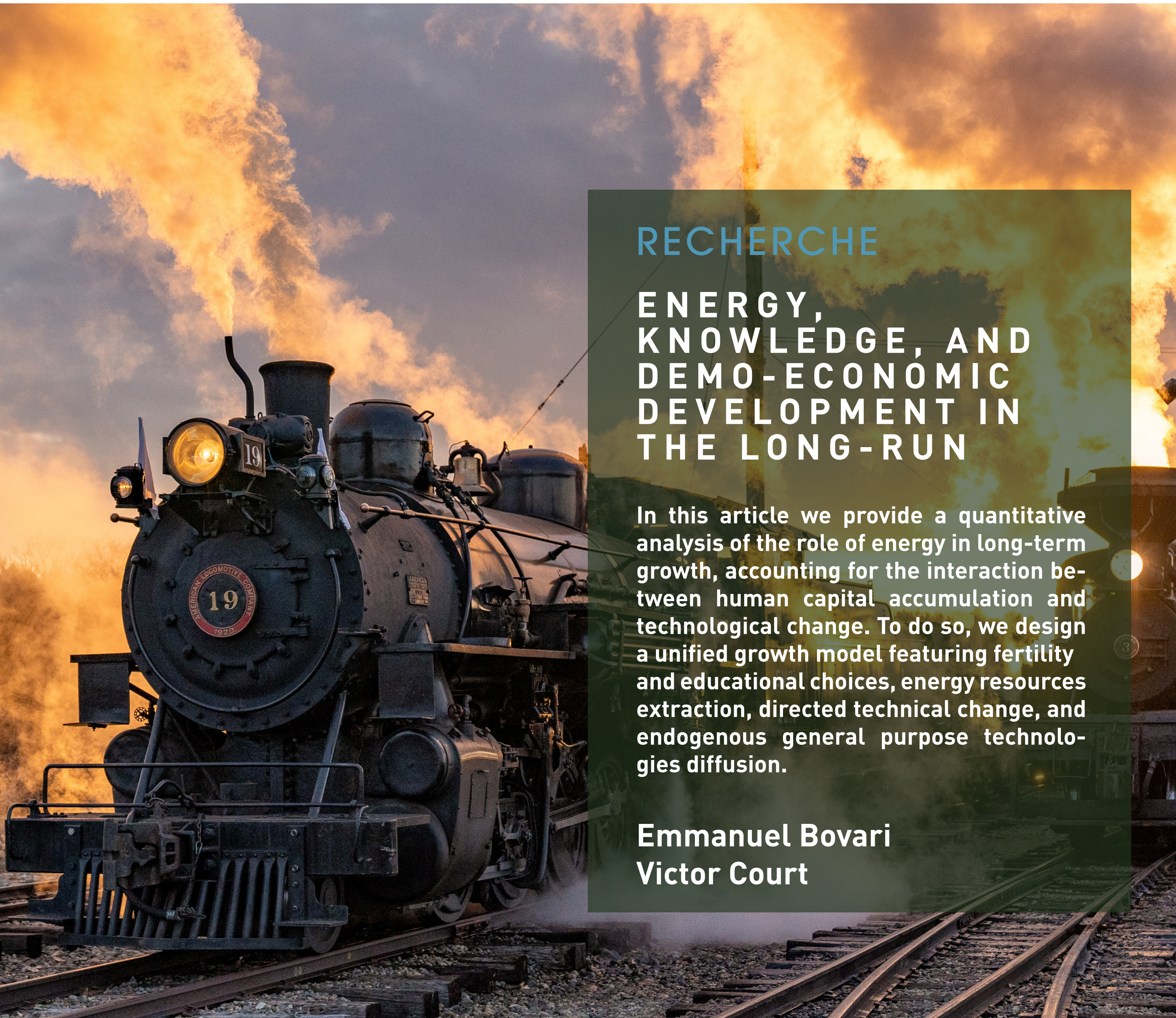

RECHERCHE

ENERGY, KNOWLEDGE, AND DEMO-ECONOMIC DEVELOPMENT IN THE LONG-RUN

In this article we provide a quantitative analysis of the role of energy in long-term growth, accounting for the interaction between human capital accumulation and technological change. To do so, we design a unified growth model featuring fertility and educational choices, energy resources extraction, directed technical change, and endogenous general purpose technologies diffusion.

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Energy, Knowledge, and Demo-Economic Development in the Long Run: A Unified Growth Model

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Abstract

Because energy is usually absent from modern growth analysis, Unified growth models designed to study the economic take-off process have tended to focus on the role of human capital accumulation and its interaction with technical change. However, prominent economic historians claim that the transition to coal and its use in steam engines was the main driver of the Industrial Revolution. In order to try to reunite these diverging point of views, we provide in this article a quantitative analysis of the role of energy in long-term growth, accounting for the interaction between human capital accumulation and technological change. To do so, we design a unified growth model featuring fertility and educational choices, energy resources extraction, directed technical change, and endogenous general purpose technologies (GPTs) diffusion. The associated energy transition results from the endogenous shortage in the availability of renewable resources (wood), and the arrival of new GPTs that, together, redirect technical change towards the exploitation of previously unprofitable exhaustible energy (coal). A calibrated version of the model replicates the historical episode of the British Industrial Revolution, for which counterfactual simulations are performed to characterize the impact of the energy transition on the timing and magnitude of the economic take-off. Another numerical exercise provides a comparative analysis of Western Europe and Eastern Asia, emphasizing the relevance of discrepancies in terms of energy resources accessibility to explain the diverging dynamics of these two world regions. Our findings show that, whenever demographic dynamics and human capital accumulation are accounted for, energy use appears as a vital catalyst for the economic take-off process.

which has been promoted by recent advances in the growth literature

Keyword: Unified Growth Theory, Directed Technical Change, Energy Transition, Demographic Transition

JEL Classification: C68, J13, J24, N10, O31, O40

1 Introduction

In this article, we assess the role of energy in long-term growth alongside the more conventional contribution of human capital, fertility choices, and technological changes. In a theoretical model that is simulated against historical data, we show that changes in energy resources and their uses acted as a catalyst for the Industrial Revolution. Our results suggest that the transition towards fossil fuel was not a necessary condition to observe an economic take-off, but it was clearly required to observe the magnitude in the take-off that was required to reach current levels of economic development. Moreover, heterogeneity in energy resources endowments help to shed some light on the timing of the economic take-off across world regions, and thus contributes to the analysis of the *Great Divergence* debate.

Precisely, we develop in the current paper a unified growth model able to account for the transition between: (i) a pre-modern organic regime defined by limited growth in per capita output, high

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fertility, low levels of human capital, prevalent learning-by-doing technical change, and rare *general purpose technology* (GPT) arrivals; and (ii) a modern fossil regime characterized by sustained growth in per capita output, low fertility, high levels of human capital, prevalent profit-motivated technical change (R&D), and increasingly frequent GPT arrivals. More precisely, we develop a general equilibrium endogenous growth model of a closed economy with five types of agents: households, final good firm, input producers, capital good producers and innovators. Households are modeled through three overlapping generations representing children, adults and retirees. The representative adult allocate her time between labor and child rearing, and her income between consumption, savings, and education expenditures. These decisions set the supply of human and physical capital, endogenously featuring a demographic transition trough a quantity-quality trade-off that represent the first economic take-off mechanism discussed in this paper. On the supply side, the production of the final good is competitive and relies on imperfectly substitutable inputs. Each input is also produced competitively, relying on human capital, capital goods, and an energy flow extracted from a sector-specific resource. Moreover, each input producer benefits from sector-specific learning-by-doing improvements – prevalent during the pre-modern era –, and Schumpeterian technical changes – that gain significant momentum during the industrial revolution because profit-motivated R&D relying on human capital becomes abundant when the demographic transition is initiated. The interaction between technical change and the relative prevalence of energy inputs features the second mechanism for economic take-off discussed in this paper: the exhaustion of pre-modern energy sources foster technical improvements in modern-energy complementary inputs, and the resulting energy transition allows to expand the production scale by exploiting additional energy resources. At last, we introduce some stochasticity in this framework by modelling the endogenous arrival of GPTs with increased frequency while applied knowledge improves, which features technical business cycles that shape the efficiency of both learning-by-doing and Schumpeterian technical changes.

We then provide some analytical results about the underlying force that shape these two economic take-off mechanisms. We finally turn to numerical simulations to replicate and analyse the historical experiences of Great Britain on the one hand, and Western Europe and Eastern Asia on the other. To do so, we calibrate our model with historical data and implement a counterfactual analysis of changes in of key factors, such as energy extraction costs, energy resource levels, and knowledge accumulation. We quantitatively show that, if energy use *per se* is not a necessary condition to reach a modern growth regime in our model, it certainly is a catalyst required to achieve high level of economic development. Moreover, variations in energy resources "quality" (extraction cost) and endowments explain the timing differential in economic take-off that resulted in the Great Divergence phenomenon. In particular, we show that a lower stock or a higher extraction cost for coal, or a lower initial level in learning-by-doing technical change, introduce significant delays in economic take-off. Hence, this paper helps to reconcile economic growth theory with historical facts regarding the role of energy and knowledge. It supports a crucial for the transition to fossil energy because such this element is necessary to reproduce the timing and magnitude, but not the occurrence *per se*, of past economic take-off.

The remaining of the article is organized as follows. Section 2 reviews the related literature and presents several empirical facts regarding the relation of demography, knowledge, and energy with the take-off towards modern economic standards. Based on these insights, Section 3 develops a knowledge-based and energy-centered unified growth model, for which Section 4 provides some analytical results. Then, Calibrations and simulations of the model with respect to the historical experiences of Great Britain, and then Western Europe and Eastern Asia, are performed in Section 5. Finally, a summary of the contributions of this article is given in Section 6.

2 Related Literature

In this section, we briefly survey the literature centered around demography, knowledge, and energy in the economic growth process. Performing this literature review allows us to precise the gap we want to fill and the main building blocks of the theoretical framework presented in Section 3.

2.1 Demography, Human Capital, Useful Knowledge and Technology

This article contributes to the unified growth theory that was pioneered by the seminal work of Galor & Weil (2000).[1] In this model and in subsequent frameworks, a central feature is the tight relationship between human capital accumulation, the endogenous demographic transition, and the take-off from limited to sustained economic growth during the industrial revolution. In some of these models, parents consciously evaluate a trade-off between the number of children they want to have and the level of education they choose for their children. However, as detailed in A.1, the empirical evidence precludes any consensus on the existence and the causes of such a child quantity-quality trade-off. Hence, several authors have emphasized that additional mechanisms are necessary to explain the timing of both the demographic transition and the industrial revolution, especially in Great Britain. Moreover, if schooling and the resulting improvements in human capital were indeed probably important to foster the *second* phase – after around 1850 – of the industrial revolution, it is not so sure that human capital of the general population was crucial before (see details in A.2).

Recent theoretical and empirical literature indeed emphasise that *useful knowledge* is a more likely cause of the intellectual changes necessary for the *first* phase of the industrial revolution. Goldstone (2009), Jacob (2014), and Mokyr (2011, 2017) attribute much of the credit for the burst of innovations and accelerated diffusion of best practices after 1750, not to mass education in general, but to the scientific culture that emerged with the European Enlightenment. They argue that Western European societies were inclined to see technical breakthroughs in the eighteenth century thanks to the increase in – and propagation during the previous two hundred years of – printing books, publishers, scientific societies, university networks, relatively accessible public lectures, and growing day-to-day exchanges between scientists, engineers, and craftsmen. Hence, these authors explain the success of the British industrial revolution through changes in the intellectual, social, and institutional background environment that enabled Great Britain to acquire a modern science culture. This change in the intellectual environment permeated the whole society and was decisive in converting *useful* knowledge – i.e., ideas and inventions that often came from distant parts of the world – into workable innovations that were rapidly transformed into practical technologies necessitating *applied* knowledge – i.e., skills – yielding profits to their developers. Recent empirical assessments support the role of useful knowledge and applied knowledge as crucial initial levers of the industrial revolution. For instance, Squicciarini & Voigtländer (2015) shows that 'upper-tail knowledge' – proxied by *Encyclopédie* subscribers' density – is a strong predictor of city growth after the onset of the French industrialization. Furthermore, de Pleijt *et al.* (2019) perform a quantitative assessment of the effect of industrialization – captured by the number of steam engines per person installed in

[1] Following the path-breaking work of Galor & Weil (2000) different mechanisms have been discussed to explain the long-term growth process, such as: the scale effect of population on technical change in Galor & Weil (2000), Yakita (2010), Galindev (2011), Fröling (2011), and Strulik *et al.* (2013); the Darwinian selection of child-quality oriented individuals in Galor & Moav (2002); the Darwinian selection of entrepreneurial-oriented individuals in Galor & Michalopoulos (2012); the improvements in gender equality in Lagerlöf (2003); the decreasing demand for child labor in Doepke (2004); the decreasing child mortality rate and consequent improvement in life expectancy at birth in Cervellati & Sunde (2005); the improvement of health (but not longevity) in Hazan & Zoabi (2006); the increasing productivity of agriculture in Strulik & Weisdorf (2008); the increasing size of markets in Desmet & Parente (2012); and the increase in general knowledge in O'Rourke *et al.* (2013).

England by 1800 – on the average working skills of 2.6 millions workers. Their findings support for a causal relationship going from the diffusion of steam engines to higher skill-demand. Moreover, **?** show that early industrialization was negatively associated with primary schooling and with the acquisition of literacy skills for women. Overall, **?**'s (**?**) findings tend to (i) confirm Mokyr's (2011) conclusion that basic education and the associated human capital was not a key ingredient in England's early industrialization; and (ii) show that the causal relationship running from useful knowledge (i.e., scientific ideas and inventions) to industrialization, highlighted by Squicciarini & Voigtländer (2015), was complemented by a causal relationship going from industrialization to applied knowledge (i.e., skills).

Recent unified growth models have taken into account these different findings. Strulik *et al.* (2013) introduce a setting where technical change is initially only due to learning-by-doing *prior* to the apparition of an expanding input variety R&D sector that then fosters sustained economic growth. O'Rourke *et al.* (2013) introduce a stock of useful general knowledge whose level impacts the cost of innovation in a Schumpeterian R&D sector. As will be shown in Section 3, we build on these articles and explicitly introduce a stock of useful knowledge. Moreover, our population module is an adaptation of Strulik *et al.*'s (2013) formulation to which we have added an impact of the stock of useful knowledge on the efficiency of the human capital production. Furthermore, following the historical and theoretical account of Lipsey *et al.* (2005), we also rely on the innovative theoretical approach of Schaefer *et al.* (2014) in which the stock of useful knowledge shapes the pattern of arrival of general purpose technology (GPT). In turn, the latter then impact the rate of change of both learning-by-doing and Schumpeterian technical changes in all sectors.

2.2 Energy and the Economic Engine

Energy use is usually absent from modern growth analysis because it is assumed to have a second-order impact due to its abundance and modest cost-share. However, as detailed in A.3 and , several empirical and theoretical arguments indicate that energy is in fact far more important for the modern growth process than usually assumed. Regarding the British economic take-off process during the industrial revolution, the central role of coal and the steam engine is obvious for many economic historians. Understanding this line of arguments requires to start our analysis a few decades before this major historical event.

First, it is recognized that, from the sixteenth century onward, the expansion of European markets due to the Atlantic (slave) trade and the institutional changes that accompanied it, lead several Western European countries towards an *Industrious Revolution* characterized by an increasing prevalence of labor offered on a specialized and relatively well-remunerated markets (de Vries, 1994). In particular, for two Western European proto-industrial nations, Great Britain and the Netherlands, wages broadly increased from the sixteenth to the eighteenth centuries relatively to other parts of Europe and the world. This so-called *Little Divergence* within Europe implied that incentives for labor-saving technologies were more important in Great Britain and the Netherlands compared to other European nations, while nonexistent in China, Japan or India where labor remained relatively cheap (Allen, 2011; Allen *et al.*, 2011). Simultaneously, because proto-industries heavily relied on wood fuel, and because the supply of wood is ultimately bounded, wood scarcities leading to price increases were frequent in most of Western Europe and especially in Great Britain (Pomeranz, 2000, pp. 220–223). There, imports of wood from the Baltic Sea and North America did not prevent its price from rising by about 700% between 1500 and 1630, much faster than general inflation (Crosby, 2007, p. 69). As Goldstone (2002, p. 361) rightly states, the ultimate bottleneck in pre-industrial economies lay not in land or other raw materials but in energy.

At these times of significant incentives for both labor-saving and woodfuel-saving technologies in Western Europe, Western European countries, and here again most notably Great Britain, were lucky to be endowed with large and relatively accessible deposits of coal, which was not the case for

China where coal deposits were distant from the major manufacturing regions of the lower Yangzi and the south (Pomeranz, 2000, p. 65). Substituting wood and charcoal for coal in heat generation had been well-known for centuries. However, a major breakthrough came with the development of the steam engine that enabled to turn the heat released from coal combustion into mechanical energy. As a provider of mechanical energy, steam engines were substitutes for waterwheels and labor. As argued by Malm (2013), steam engines progressively replaced waterwheels not because they were initially more powerful, more efficient, cheaper or because the hydraulic potential of British rivers was scarce, but simply because they were mobile and could work continuously within cities where abundant and concentrated quantities of labour were present after agricultural improvements had pushed many peasants towards cities. On the contrary, waterwheels were fixed, located in rural environment, and were prone to stop due to flooding or freezing of rivers. Steam engines enabled a large concentration of energy in time and space, were eventually more powerful than previous sources of mechanical energy, and turned out to be much easier to control (Kander *et al.*, 2013, p. 367).[2] Kander *et al.* (2013, pp. 367–368) assert that the high complementary between coal, the steam engine, and the iron industry was crucial in delivering unprecedented amounts of mechanical energy that structurally reshaped the industrializing British society. As Berg (1994, p. 207) similarly notes, it was not the spinning machinery itself (in operation since 1770) that made England a leader in textile production, but rather the application of steam power to spinning, water and surface transport, brick- and iron-making, grain-threshing, construction, and all other sorts of manufacturing processes that transformed the British economy.[3]

Synthesizing those elements, Allen (2011, 2009) comprehensively argues that the British Industrial Revolution originated in the willingness and ability of its people to (i) tap their favorable coal endowment thanks to economic incentives represented in relative factor prices (of labor, capital, wood, and coal, see Figure 1), and (ii) apply knowledge brought by science (as already highlighted in Subsection 2.1) to convert coal into mechanical energy, and in doing so direct and foster sustained technical change during the Industrial Revolution. Exploiting geographical variation in city and coalfield locations, alongside temporal variation in the availability of coal-powered technologies, Fernihough & O'Rourke (2014) support and quantify Allen's (2011; 2009) theory. Precisely, these authors estimate that coalfields' location and the availability of steam engines explain around 60% of the growth in European city populations from 1750 to 1900. In a complementary approach, Malanima (2016, pp. 96–99) estimates land- and labor- savings due to coal use, respectively as a source of heat and mechanical energy, in England & Wales on the period 1560–1913. The results indicate that from 1800 to 1900, the land-related (resp. labor-related) savings grew from 1 to 14 times the extent of the entire country of 15 million hectares (resp. from 1 million to almost 290 million workers while the English labor force was about 13–14 million workers in 1900). These estimates strongly support Wrigley's (2016, pp. 2–4) claim that, given the amount of energy required to produce iron and steel on a large scale, an Industrial Revolution could not have been accomplished as long as mechanical energy continued to be provided principally by human and animal muscle – and thus ultimately by the annual flow of solar energy derived from plant photosynthesis and river flows.

Based on these evidences, we hereafter present a unified growth model that allow us to (i) distinguish several energy forms (and their corresponding prices), (ii) describe the technical change associated with fossil energy as a response to pre-modern renewable energy shortage, and (iii) emphasize the key role of the abundant and cheap fossil energy supply during the Industrial Revolution and the subsequent modern regime.

[2] It is sometimes argued that early steam engines were extremely inefficient and that the end-use cost of the mechanical energy produced by a steam engine was not lower than that produced by a windmill, waterwheel, or worker. This paradox is solved when one considers that a steam engine is both mobile and continuously operational.

[3] It is because they completely miss the importance of the synergy between coal, the steam engine, and the iron industry that Clark & Jacks (2007), relying solely on the contribution of coal mining rents to national income, can claim that coal made a negligible contribution to the success of the British Industrial Revolution.

Fig. 1 Panel (a) - Labor to capital real prices in England *(solid line)*, Strasbourg *(dashed line)*, and Vienna *(dotted line)*, 1630–1800; Panel (b) - Real prices of firewood *(solid line)* and coal *(dashed line)* in England, 1300-1870, time series smoothed with Hodrick-Prescott filter (factor 1000); Panel (c) - Labor to energy real prices in six cities, early 1700s.

Data sources: (Allen, 2009, p. 139-140) for panels (a) and (c) and Fouquet (2011) for panel (b).

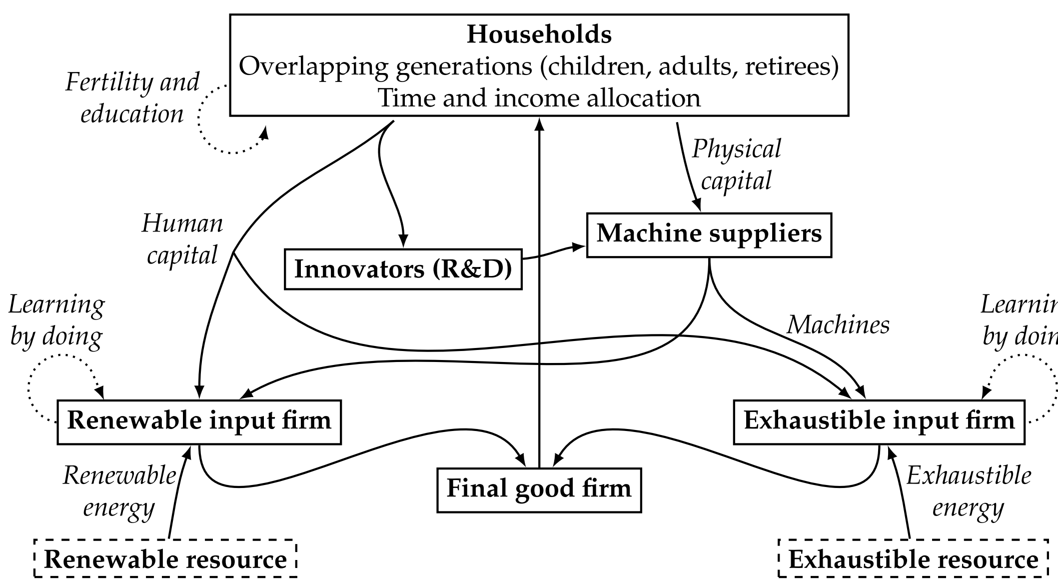
2.3 Recent Long-Term Growth Models

The role of energy use in long-term growth has not been extensively discussed in the unified growth literature. As a matter of fact, the only published exception is the article of Fröling (2011) and Gars & Olovsson (2019). However, Fröling's (2011) set-up does not disentangle the different incentives that shape the synergy between technologies and energy resources, which explains why Fröling's (2011) model fits rather poorly to the global historical data. Several papers of the endogenous growth literature have tried to better address the interplay between resources, technologies and growth. In his seminal article, Acemoglu (2002) focuses on the interplay between directed technical change and growth patterns that appear when R&D is profit-motivated and complement specific technologies. Acemoglu *et al.* (2012) apply this framework to climate change issues, introducing an exhaustible resource that shape R&D bias. Additional contribution have generalized this approach to many sectors (Gars & Olovsson, 2019; Lemoine, 2018), or to the specific case of the British Industrial Revolution as in Otojanov (2018) and Stern *et al.* (2019). These papers assume either constant energy extraction, and/or a normalized pool of scientist driving R&D, which preclude any population scale effect. Moreover, the population size is exogenous in all these approaches that neither incorporate education nor human capital. As a consequence of these omissions – on which unified growth models usually put an important emphasis –, these contribution might overestimate the role of changes in energy use during the Industrial Revolution. Indeed, the coal-hypothesis appears as the sole driver of the industrial revolution in these approaches. Therefore, the present article complements this literature through its focus not only on: (i) the interaction between technology and extraction costs that allows for learning effects documented in the literature (see e.g., Court *et al.*, 2018), but also on (ii) Schumpeterian R&D fueled by human capital, and (iii) endogenous fertility and education choices. Our modeling setting allows us to disentangle the role of energy and human capital accumulation for the timing and magnitude of the economic take-off. Moreover, we extend our numerical analysis to the analysis of the British case and the comparative development of Western Europe and Eastern Asia, as an assessment of the narrative prompted by Pomeranz (2000) and Allen (2011, 2009). Our model shows that once fertility and human capital are accounted for, energy is not the sole root-cause of the Industrial Revolution, but it remains a key catalyst needed to explain the different paces of development trajectories over the last two centuries.

3 A Model of Energy, Demographic and Economic Transitions

In this section, we construct a general equilibrium growth model to study the role of energy use in the economic take-off during the industrial revolution. We consider a closed economy in discrete time, indexed by t. We distinguish five types of agents: household, final good producer, input producers, capital good producers and innovators. The representative household consists in three overlapping generations: childhood, adulthood and retirees. Only adults allocate their workforce between labor and rearing children, and their income between present consumption, savings for (future consumption while retired), and children education. These decisions endogenize the provision of human and physical capital. The final good producer combines inputs to provide a composite good that will be used for consumption and investment. Input producers, combine labor (human capital), capital goods, and an energy input coming from a specific resource for each input producer – e.g. renewable and exhaustible. Monopolistic capital good producers transform physical capital into machines to supply intermediate producers. Innovators hire labor to perform R&D and, in case of innovation success, replace an existing monopolist in the corresponding sector by providing a more efficient machine. Thus, input sectors differ in terms of energy resource availability and average technical level. This affects not only the current production in the final good sector through substitution effects, but also the future production through endogenous effort to perform sector-specific R&D. At last, R&D being a quite recent way of technical progress that gained momentum during the industrial revolution, we also allow for sector-specific learning-by-doing effects that will prevail during the pre-industrial era. Figure 2 graphically represents our modeling framework. Finally, we introduce some stochasticity in this setup by modelling the endogenous arrival of GPTs, with increased frequency while applied knowledge improves, featuring technical business cycles that shape the efficiency of learning-by-doing and Schumpeterian technical changes in all sectors.

Fig. 2 Graphical representation of the economy.



3.1 Preferences and Factor Supply

We consider an economy in which each overlapping generation (children, adults, and retirees) admits a representative individual. At each time period t, there are N_t representative adults. Each representative adult is endowed with one unit of time that can be dedicated to (i) effective labor, to earn a wage, or (ii) child-rearing, thus setting the number of adults in the next period, $t + 1$.

Moreover, the representative adult allocates her income between direct consumption, children's education, and savings. Children do not work, nor consume, and inherit from the level of education chosen by adults. Thus, higher expenditures in education builds greater human capital for the next generation, hence greater effective labor supply. Retirees do not work and consume their past savings. Fertility, education, and savings decisions shape the supply of human and physical capital.

3.1.1 *Household's Preferences*

In order to derive the main results conveniently and get closed-form solutions, we make a number of simplifying assumptions that are typical for unified growth overlapping generation models, that admit a representative household at each generation (e.g. Strulik *et al.*, 2013).

Assumption 1 *Each household consists in: (i) a number $N_t \geq 0$ of unisex parent with non-operational education expenditures motives, (ii) a continuous number $b_t \geq 0$ of children, and (iii) a number $N_{t-1} \geq 0$ of retirees.*

These assumptions respectively allow to avoid matching issues between adults, indivisibility problems for children, and issues of dynastic value function maximisation regarding education.[4] The representative adult is the only decision-making unit and admits perfect expectations. Her preferences are represented by a logarithmic utility function defined over: (i) immediate consumption, $c_t \geq 0$, (ii) future consumption during retirement, $c^r_{t+1} \geq 0$, (iii) births per capita, $b_t \geq 0$, determining family size, and (iv) the future level of human capital, $h_{t+1} \geq 0$, that each child receives through present education. Thus, the representative household's utility function writes

$$u_t = \log(c_t) + \chi \log(c^r_{t+1}) + \rho \log(h_{t+1}) + \eta \log(b_t), \tag{1}$$

where the positive parameters χ, ρ, and η capture, relative to current consumption, the elasticities of utility with respect to future consumption during retirement, the human capital of children, and the family size.

The representative adult benefits from two revenue sources: labor wages and patenting revenue from the R&D sector. The latter is driven to zero under the free-entry condition in R&D and our market design for capital goods production presented hereafter in Subsection 3.2.3.[5] We adopt a sequential trading approach of the young adult's decisions. At each time period, there is a spot market for the final good (that is either consumed, saved or spent in education). In addition, there is an asset market though which savings decision are performed, one unit of consumption invested in period t delivers $1 + r_{t+1}$ accrued units of consumption in period $t + 1$. Thus, the representative adult planner face the two following constraints:

$$\begin{aligned} c_t + s_t + b_t e_t &\leq w_t h_t [1 - \tau b_t], & (2)\\ c^r_{t+1} &\leq (1 + r_{t+1}) s_t, & (3) \end{aligned}$$

where $s_t \geq 0$ is the savings of generation t, $e_t \geq 0$ is the education expenditure per child,[6] $w_t \geq 0$ is the competitive market wage per efficiency labor unit $h_t \geq 0$, $\tau \geq 0$ is the fraction of the adult's time endowment required to raise one child, and $r_{t+1} \geq 0$ is the interest rate from period t to period $t + 1$. Eq. (2) represents the current period budget constraint of the representative adult,

[4]This last point means that the parent's motivation to spend on their children's education is not driven by the anticipation of the increase of children's utility caused by this expenditure, but by a general desire for having 'higher quality' children.

[5]Relaxing these hypotheses would not change qualitatively the behavior of the representative household.

[6]In other words, the provision of aggregate education services $b_t e_t N_t$ has a cost of $b_t e_t N_t$ units of final good. This technical assumption ensures market clearing on the final good market, as illustrated hereafter in Eq. (26).

the income (right-hand side) being allocated to current consumption, savings and education (left-hand side). Raising one child can be seen as an opportunity cost valued at $\tau w_t h_t$. Eq. (3) represents the intertemporal budget constraint, the sole revenue of the current retired generation consisting in the savings accrued in former adulthood, as is typical in the overlapping generation literature (see de La Croix & Michel, 2002). Formally, we shall assume that savings are invested in a financial asset at the interest rate r_{t+1}. Relying on the usual 'no arbitrage' assumption between the financial market and the physical market—physical assets being used for production—, this interest rate shall then be equated to the price charged to productive firms *plus* the depreciation rate of physical capital. For simplicity and to avoid multiplying notation, we immediately assume for the rest of this paper that savings are only invested in physical capital priced accordingly.

Furthermore, we assume that education expenditures, e_t, are converted into human capital h_{t+1} through a schooling technology that controls for schooling costs, approximated by w_t,[7]

$$h_{t+1} = A_E(Q_t)\frac{e_t}{w_t} + \bar{h}, \tag{4}$$

where $\bar{h} \geq 0$ represents informal human capital acquired without formal education,[8] and $A_E(\cdot)$ a $\mathcal{C}^2$ function of the total applied knowledge of the economy, $Q_t \geq 0$, defined thereafter. We assume that the education technology shows decreasing returns with respect to this stock of applied knowledge, so $A_E(\cdot)$ satisfies the following conditions: $A_E(0) = 0$, $\partial A_E/\partial Q > 0$ and $\partial^2 A_E/\partial Q^2 < 0$. In other words, we assume that the overall level of applied knowledge, Q_t, is a good proxy for various phenomena that positively affect the efficiency of schooling, such as the rising spatial density of schools (Boucekkine *et al.*, 2007), the evolution of social norms favoring formal education, or changes in law limiting child's labor (Doepke, 2004). Another interpretation, which is more consistent with the seminal work of Galor & Weil (2000), is that a greater stock of global knowledge increases the returns to education to cope with a potentially more complex and rapidly evolving environment. Our specification is neutral with regards to these two interpretations. Throughout this paper, we assume $A_E(Q_t) = \bar{A}_E Q_t/(1+Q_t)$, with $\bar{A}_E > 0$.

The utility-maximizing behavior of the representative households can then be formally introduced through the following optimization problem.

Problem 1 (HH - Household) *The representative adult planner seeks to maximize the utility function define in Eq. (1) under the schooling technology defined in Eq. (4), and the budget constraints defined in Eqs (2) and (3). Hence, taking factor prices w_t and r_t as given, the household's problems writes*

$$\begin{aligned}\max_{c_t, c^r_{t+1}, s_t, b_t, e_t} \quad & u_t = \log(c_t) + \chi \log(c^r_{t+1}) + \rho \log(h_{t+1}(e_t, Q_t)) + \eta \log(b_t) \\ s.t. \quad & c_t + s_t + b_t e_t \leq z_t[1 - \tau b_t], \\ & c^r_{t+1} \leq (1 + r_{t+1}) s_t, \\ & c_t \geq 0, c^r_{t+1} \geq 0, s_t \geq 0, b_t \geq 0, e_t \geq 0.\end{aligned}$$

3.1.2 Supply of Physical and Human Capital

Aggregating over the number of representative adults, N_t, at each period, households inelastically supply two production factors that depend on their savings and fertility decisions: physical capital

[7] One can think of w_t as the wage of teachers, hence e_t/w_t represents efficient education expenditure.

[8] Such a basic human capital level can be thought as informal knowledge that children acquire through the time τ spends observing and imitating their parents and peers at work. This knowledge (of farming or a particular craft, for example) is useful, i.e. it creates human capital at level $\bar{h}$, but it comes for free, at no educational cost. On the contrary, e_t is a financial investment that allows the child to receive a formal education through school and the consumption of cultural goods in order to increase their human capital above $\bar{h}$.

and human capital. The stock of physical capital results from the aggregate accumulation of net savings of households following the usual law of accumulation

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{5}$$

where $\delta \in [0,1]$ stands for the depreciation rate of capital, and $I_t \geq 0$ is the net investment which consists in savings of the current generation of young adults, $s_t N_t$, *minus* the dissavings of the current generation of retired adults. As we preclude voluntary bequest motives, the retired generation withdraws all the remaining stock of physical capital they hold, that is $(1-\delta)K_t$.[9] The stock of physical capital for the next period is thus entirely determined by the savings of the current young adult generation, such that Eq. (5) simply write $K_{t+1} = s_t N_t$. Turning to the supply of human capital, one can first derive the law of motion of the population of young adults, N_t, as

$$N_{t+1} = b_t N_t.^{10} \tag{6}$$

Taking child-rearing time into account, the size of the workforce, L_t, is then given by

$$L_t = (1-\tau b_t)N_t, \tag{7}$$

whereas the aggregate human capital supply, H_t, corresponds to

$$H_t = h_t L_t. \tag{8}$$

3.2 Production

We turn now to the description of the production side of the economy. Following Acemoglu *et al.* (2012), we consider a final sector in which a composite good is competitively produced with k imperfectly substitutable inputs. Each k input comes from the combination of intermediate capital goods (i.e., machines), human capital, and primary energy. The different k energy types come either from renewable or exhaustible resources, and their use incurs a sector-specific extraction cost.

Final good and inputs production are assumed to be perfectly competitive, while the supply of machines is monopolistic. Indeed, technical change is assume to take place in a Schumpeterian fashion where quality improvements are specific to each machine line (Aghion & Howitt, 1992). Thus, machines are provided by monopolists owning a patent on their variety, endogenously supplanted by successful innovators in a process of creative destruction. Moreover, as described in Section 3.3, this profit-motivated R&D innovation interacts with General Purpose Technologies (GPTs), which also shapes the level of a learning-by-doing knowledge, affecting production.

3.2.1 Final Composite Good

The final composite good, Y_t, is an aggregate of $k \in \mathcal{K}$ inputs, $Y_{k,t}$, according to a Constant Elasticity of Substitution (CES) technology,

$$Y_t = \left[\sum_{k\in\mathcal{K}} \vartheta_k Y_{k,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{9}$$

where $\sum_{k\in\mathcal{K}} \vartheta_k = 1$, with $\vartheta_k \geq 0$ measuring the relative economic usefulness of inputs, Y_k, and $\sigma \geq 0$ is the elasticity of substitution between inputs. The set $\mathcal{K}$ refers to the kind of primary energy used

[9] It is worth mentioning that, as exposed hereafter in Subsection 3.2.3, physical capital holders are exactly compensated for the depreciation δ, such that the overall interest paid to the retired generation is the interest rate r_t plus the depreciation rate δ.

[10] Accordingly, the total population, P_t, corresponds to $P_t = 2[(1+b_t)N_t + N_{t-1}]$.

to produce the corresponding input, as defined shortly. This good is used for consumption and investment purposes by households, and her price taken as the numeraire. The final good sector can then be formally described through the following optimization problem.

Problem 2 (FG – Final good producer) *The final good producer is perfectly competitive and uses the technology defined in Eq. (9). Its price is chosen as the numeraire. The representative firm takes the prices of the final inputs* $\{p_{k,t}\}_{k\in\mathcal{K}}$ *as given to solve*

$$\max_{\{Y_{k,t}\}_{k\in\mathcal{K}}} \left[\sum_{k\in\mathcal{K}} \vartheta_k Y_{k,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \sum_{k\in\mathcal{K}} p_{k,t} Y_{k,t}$$
$$s.t.\ \ \forall k \in \mathcal{K}, Y_{k,t} > 0.$$

3.2.2 Inputs

We denote by $\mathcal{K}$ the set of primary energy resources that are specific to each input sector. These energy carriers are either: (i) renewable energy resources corresponding to biomass, wind, water and solar flows; or (ii) exhaustible energy resources corresponding to fossil fuels – such as coal, oil, gas – and fissile materials – such as uranium. For each primary energy type, $k \in \mathcal{K}$, the provision of the final input, $\{Y_{k,t}\}_{k\in\mathcal{K}}$, results from the combination of (i) a continuum of machines of measure one, $\{x_{k,i,t}\}_{i\in[0,1]}$ with specific endogenous quality, $\{q_{k,i,t}\}_{i\in[0,1]}$, (ii) human capital, $H_{k,t}$, and (iii) a primary energy flow, $E_{k,t}$. It is worth mentioning that $q_{k,i,t}$ designates the quality level *after* innovation decisions of period t, which occur *prior* to production decisions. These factors are combined according to the following Cobb-Douglas technology,

$$Y_{k,t} = A_{k,t}\left[\int_0^1 q_{k,i,t}^{1-\alpha_k} x_{k,i,t}^{\alpha_k} di\right] H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}, \tag{10}$$

where $\alpha_k + \beta_k + \gamma_k = 1$, and $A_{k,t}$ is the technical level achieved through learning-by-doing in sector k. The provision of the final input $\{Y_{k,t}\}_{k\in K}$ is perfectly competitive. Each final input is ultimately sold, at a price $p_{k,t} > 0$, to the final composite good sector. The costs of sector-specific machines, human capital and sector-specific energy flow are respectively denoted by $p^x_{k,i,t} > 0$, $w_t > 0$, and $\Psi_{k,t} > 0$. The latter cost consists in a function $\Psi(\cdot,\cdot)$ that captures the assumed convexity of the extraction of primary energy. [11] We model two underlying forces that shape this cost function, namely, the remaining level of primary energy resource, and the level of technical advancement achieved in the sector. The dynamics of primary energy resources are given by

$$\mathcal{R}_{k,t} = \mathcal{R}_{k,0} - E_{k,t}, \tag{11}$$

in case of a renewable resource, and

$$\mathcal{R}_{k,t} = \mathcal{R}_{k,0} - \sum_{a<t} E_{k,a}, \tag{12}$$

[11] This extraction cost might also be seen as the price charged by a perfectly competitive primary energy extracting firm. Thus, our framework is neutral regarding an integrated or segmented energy sector.

in case of an exhaustible resource. The set $\{\mathcal{R}_{k,0}\}_{k\in\mathcal{K}}$ is determined by the natural environment and corresponds to the levels of (renewable or nonrenewable) primary energy virgin resources.[12] Following Court *et al.* (2018), we suppose that as each stock of resource $\mathcal{R}_{k,t}$ gradually decreases towards zero, it becomes increasingly difficult to extract primary energy. On the other hand, technical improvements can lower the extraction cost of primary energy. We use the stock of applied knowledge specific to the sector, $\hat{Q}_{k,t}$ (introduced thereafter in Eq. (17) of Section 3.3), as a proxy for the technical advancements that decrease extraction costs, so as to extend the amount of economically profitable *reserves* out of physically bounded *resources*. Normalizing $\mathcal{R}_{k,t}$ and $\hat{Q}_{k,t}$ to their initial values, we then define $\{\Psi_k\}_{k\in\mathcal{K}}$ as follows

$$\Psi_k = \overline{\Psi}_k \left(\frac{\mathcal{R}_{k,t+1}}{\mathcal{R}_{k,0}}\right)^{\psi_{\mathcal{R},k}} \left(\frac{\hat{Q}_{k,t}}{Q_0}\right)^{\psi_{Q,k}}. \tag{13}$$

with $\overline{\Psi}_k > 0$ a scaling parameter, and $\psi_{\mathcal{R},k} < 0$ and $\psi_{Q,k} < 0$ defining the convexity of the extraction cost. Input provision sectors can then be formally described through the following optimization problem.

Problem 3 (IG – Intermediate goods) *The input provider $k \in \mathcal{K}$ is perfectly competitive and uses the technologies defined in Eq. (10) and (13). The representative firm takes prices of machines, human capital, and final inputs ($\{p^x_{k,i,t}\}_{i\in[0,1]}$, w_t, and $\{p_{k,t}\}_{k\in\mathcal{K}}$ respectively), the current level of technologies, $A_{k,t}$ and $\{q_{k,i,t}\}_{i\in[0,1]}$, as given, as well as the current level of resource, $\mathcal{R}_{k,t}$, to solve*

$$\begin{aligned} \max_{\{x_{k,i,t}\}_{i\in[0,1]}, H_{k,t}, E_{k,t}} \; & p_{k,t} A_{k,t} \left[\int_0^1 (q_{k,i,t} x_{k,i,t})^{\alpha_k} di\right] H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k} \\ & - \int_0^1 p^x_{k,i,t} x_{k,i,t} di - w_t H_{k,t} - \Psi_{k,t} E_{k,t} \\ s.t. \;\; & \forall i \in [0,1], x_{k,i,t} > 0, H_{k,t} > 0, E_{k,t} > 0. \end{aligned}$$

3.2.3 Capital Goods

The machines that are used by intermediate input producers can be seen as capital goods produced from the stock of row physical capital, K_t. They are supplied under a hybrid monopolistic competition, contingent on R&D processes. As described in more detail in Subsection 3.3.3, R&D is assumed to be specific to each machine line and to occur *prior* to production decisions. As is customary in the Schumpeterian literature (see Acemoglu, 2009), and under Assumption 2 introduced shortly, innovation is assumed to be performed only by potential entrants that have greater incentives than incumbents, due to a monopoly profit they can benefit in case of success. Indeed, each successful innovator is endowed with a one-period patent on the corresponding machine line. He will then replace the former machine producer and act as a monopolist on this machine variety. Otherwise, the

[12] Nonrenewable and renewable primary energies are both physically bounded by the finite character of planet Earth. This point is straightforward for nonrenewable energies that come from finite stocks. The untapped level of a nonrenewable primary resource, $\mathcal{R}_{k,0}$, formally corresponds to the Ultimately Recoverable Resource (URR). According to British Petroleum (2015), the URR is an estimate of the total amount of a given resource that will ever be recovered and produced. It is a subjective estimate in the face of only partial information. Whilst some consider URR to be fixed by geology and the laws of physics, in practice estimates of URR continue to be increased as knowledge grows, technology advances and economics change. The URR is typically broken down into three main categories: cumulative production, discovered reserves and undiscovered resource. Renewable energies are also bounded by the ultimate size of their annual flows (as an illustration, one might consider that, for a given year, the maximum solar energy ultimately recoverable cannot exceed the natural sun radiation), which is called the Technical Potential (TP) and corresponds to $\mathcal{R}_{k,0}$ in our framework. For the IIASA (2012, chapter 7, p. 434), the renewable Technical Potential is the degree of use that is possible within thermodynamic, geographical, or technical limitations without a full consideration of economic feasibility.

machine line is produced competitively under the previous design, a situation one shall interpret as patents becoming public. This hybrid setup, combining perfect and monopolistic competition with a free-entry condition into R&D introduced in Eq. (25), ensures that there are no aggregate monopoly profits.[13]

For each machine line i in the sector k, the production technology is linear and transforms one unit of the capital stock, K_t – which is rented from households at the interest rate r_t *plus* the depreciation rate of capital δ – into one unit of specialized machine $x_{k,i,t}$. Hence, the corresponding operating profit is $\pi_{k,i,t} = (p^x_{k,i,t} - r_t - \delta)x_{k,i,t}$. The capital good provision can be formally described through the following optimization problem.

Problem 4 (CG – Capital good producers) *Each capital good sector $k \in \mathcal{K}$ sustains a hybrid regime of perfect and monopolistic competition, depending on the success of profit-motivated R&D. Whenever innovation is unsuccessful in machine line i, the latter is produced under perfect competition. In the first case the machine price is equated to the marginal cost of production, that is $p^x_{k,i,t} = r_t + \delta$ (also denoted $p^c_{k,i,t}$). Whenever innovation is successful in machine line i, the latter is produced under monopolistic competition. In the second case the capital good producer takes as given the price of capital, $r_t + \delta$, to solve*

$$\max_{p^x_{k,i,t} > 0} \pi_{k,i,t} = (p^x_{k,i,t} - r_t - \delta)x_{k,i,t}(p^x_{k,i,t})$$

thus setting the price of intermediate capital goods also denoted $p^m_{k,i,t}$.

3.3 Knowledge and Technical Change

Following Strulik *et al.* (2013), we consider two kinds of technical changes for each input provider: non-profit motivated learning-by-doing and profit-motivated R&D. As suggested by Schaefer *et al.* (2014), all these technical advancements should be interrelated by the evolution of a General Purpose Technology (GPT). Lipsey *et al.* (2005, p. 98) define a GPT as a single generic technology, recognizable as such over its whole lifetime, that initially has much scope for improvement and eventually comes to be widely used, to have many uses, and to have many spillover effects.[14]

3.3.1 General Purpose Technologies

Following Schaefer *et al.* (2014), we assume that successive vintages of GPTs, $G_{v,t}$, are developed endogenously as a result of non-profit motivated activities. In the following setting, we suppose that the level of the current GPT vintage positively affects technical change, i.e. the rate of growth of technical levels achieved through learning-by-doing or R&D. In a sense, GPTs gather all kinds of technical externalities fostering technical change. We assume that several vintages of GPTs, indexed by v, succeed over time. While it is still active, the level of a given vintage of GPT might also evolve over time, featuring learning-effects. The magnitude of the latter phenomenon, along with the evolution of the expected duration before the arrival of a new GPT, has a crucial impact on the degree of complementarity between past and current knowledge. Together, those features allow compliance with two stylized facts regarding the historical arrival of GPTs: (i) the initially

[13] These assumptions prevent any issue of inter-temporal patent allocation and pricing, without precluding the set of incentives central to profit-motived R&D (Acemoglu, 2002; Aghion & Howitt, 1992, 1998).

[14] Lipsey *et al.* (2005, p. 97) further stress that GPTs are typically use-radical but not technology-radical, meaning that GPTs do not stand out from other technologies because of a revolutionary technical basis, but rather because of outstanding applications and adaptations to other technologies and sectors of the economy. GPTs are typically not born in their final form, so they often start off as something we would never call a GPT and then develop into something that transforms an entire economy. The considerable scope of improvement of GPTs is explored as their range and variety of use increase, which in the meantime generate knowledge and practical spillovers on other technologies and organizational processes.

slow evolution of the efficiency of new GPTs, and (ii) the decreasing time interval between successive GPTs. We differ from Schaefer *et al.* (2014) in two ways. First, following the endogenous growth literature centered on human capital (Jones, 1995; Strulik *et al.*, 2013), we consider researchers (i.e., human capital allocated to R&D) rather than machines (i.e., lab-equipment purchased though financial expenditures) to be the key driver of innovation processes. Moreover, we consider that all kinds of technical changes are involved in the evolution of GPTs. Finally, we assume that GPT vintages active for period t evolve *prior* to innovation and production decisions; that is, at the beginning of the current period.

To start with, let's assume that new GPT vintages follow a non-homogeneous Poisson process with endogenous mean, μ_t, defined as

$$\mu_t = \mu_0 G_{v,t-1}, \tag{14}$$

with $\mu_0 \in [0, 1]$. The level of the active GPT, $G_{v,t-1}$ eases the arrival of the new vintage.[15] Once discovered, a new GPT is initialized with a level of

$$G_{v+1,t} = \bar{G}\tilde{q}_t \tag{15}$$

where $\bar{G}$ is a positive scaling parameter, and $\tilde{q}_t$ is an index of applied knowledge available for the GPT in period t. To further characterize the index of available applied knowledge, $\tilde{q}_t$, let us define two stocks of knowledge, $\tilde{Q}_{v+1,t}$ and $\tilde{Q}_{v,t}$. The former represents the improvement history of the current (and potentially newly introduced) GPT, whereas the latter tracks the improvement history of all previous vintages of GPTs. We now introduce the stock of applied knowledge, as an aggregate quality index of the economy

$$Q_t = \sum_{k \in \mathcal{K}} \hat{Q}_{k,t}, \tag{16}$$

with its sectoral components,

$$\hat{Q}_{k,t} = A_{k,t} + Q_{k,t}. \tag{17}$$

This index measures the extent of all technical developments through both learning-by-doing and R&D within the period t. One can then write the following identity

$$\tilde{Q}_{v+1,t} + \tilde{Q}_{v,t} = Q_t. \tag{18}$$

It is worth noting that the total quality index of the economy, Q_t, should be interpreted as the overall stock of *applied* knowledge already introduced in the efficiency of the schooling technology, Eq. (4), and the extraction technology of primary energy inputs, Eq. (13). Following Mokyr (1990, 2011), *applied* knowledge, taking the form of learning-by-doing and R&D technologies, shall be distinguished from *useful* knowledge contained in human capital and in GPT's waves that evolve concomitantly with the development of applied knowledge. The index of applied knowledge, Q_t, only captures the quantity of applied knowledge that can be used to strengthen the current GPT vintage. Depending of the complementarity between past and current applied knowledge, quantified by the parameter $\zeta \in [0, 1]$,[16] one can then define the index of applied knowledge as

$$\tilde{q}_t = \tilde{Q}_{v+1,t}^{\zeta} \tilde{Q}_{v,t}^{1-\zeta}. \tag{19}$$

As long as it remains active, the quality increments of GPT vintage evolves over time, that is as a function of the quality index $\tilde{q}_t$,

$$G_{v,t} = G_{v,t-1} + \xi \tilde{q}_t, \tag{20}$$

[15] In other words, the time interval between successive GPT vintages, T, is given by the cumulative distribution: $\mathbb{P}(T \leq t)$ where T follows a Poisson process of mean μ, hence the average waiting time corresponds to $\mathbb{E}(T) = 1/\mu$.

[16] Past (respectively current) knowledge is useless whenever $\zeta = 1$ ($\zeta = 0$).

with ξ a constant controlling the speed of *diffusion* of the current GPT vintage within the economy through the learning-by-doing and R&D performed with this vintage. Thus, the efficiency of each kind of technical knowledge is improved (as specified thereafter), again strengthening the level of the current GPT according to Eq. (18). Moreover, as GPTs are improved, the time interval before the arrival of a new GPT decreases according to Eq. (14).

3.3.2 *Learning-by-doing*

We model the technical level achieved through learning-by-doing in input sector $k \in \mathcal{K}$, $A_{k,t}$, as a function of (i) the current human capital stock allocated to the specific production sector, $H_{k,t}$, and (ii) the current GPT's level, $G_{v,t}$, capturing the conventional technical externality (i.e., the so-called standing-on-giants-shoulders effect).[17] With $\Omega > 0$ representing the efficiency with which useful knowledge contained in human capital and GPT-related know-how are converted into applied learning-by-doing knowledge for production, we have

$$A_{k,t+1} - A_{k,t} = \Omega H_{k,t}^{\omega_{H,k}} G_{v,t}^{\omega_{G,k}}, \quad k \in \mathcal{K}. \tag{21}$$

We suppose that there are decreasing returns in both human capital and technical externalities, i.e., $\omega_{H,k} \in]0,1[$ and $\omega_{H,k} \in]0,1[$, meaning that in the long run there is no more technical change through leaning-by-doing. This assumption calls for another source of technical change, namely, profit-motivated R&D presented below, to sustain growth in the long-run. Finally, we define the growth rate of the technical level (i.e., the technical change) obtained through learning-by-doing as $g_{A_k,t} \equiv \frac{A_{k,t} - A_{k,t-1}}{A_{k,t-1}} = \Omega H_{k,t-1}^{\omega_{H,k}} G_{v,t-1}^{\omega_{G,k}} A_{k,t-1}^{-1}$, with $k \in \mathcal{K}$.

3.3.3 *Profit-motivated R&D*

R&D is assumed to be profit-motivated, as customary in the endogenous growth literature (Acemoglu, 2002; Aghion & Howitt, 1992, 1998). As described in Subsection 3.2.3, in each sector, intermediate capital goods (i.e., machines) are produced either in perfect competition whenever the corresponding patent is public, i.e. R&D was unsuccessful, or by monopolists that are former successful innovators. We assume that each machine line follows a specific quality ladder: the quality of the machine line in a specific sector writes $q_{k,i,t} = q_k^{\kappa_{k,i,t}}$, where $\kappa_{k,i,t}$ is the number of successful innovations for machine line i in sector k up to time t and $q_k > 1$ the sector-specific rung of the corresponding quality ladder. At the beginning of each period, successful innovations bring the corresponding machines to a higher rung of the specific quality ladder, that is $\kappa_{k,i,t}$ becomes $\kappa_{k,i,t} + 1$. Otherwise, the quality of machines remains constant. Moreover, to derive analytical tractable results, we hypothesize that innovation is drastic, which is a customary assumption in the patent-race literature.

Assumption 2 *The innovation regime is drastic: technical improvements resulting from a successful innovation (e.g., the size of each rung in the quality ladder, q_k) are large enough such that entrants replace incumbents.*

In other words, we assume that productivity gains are large enough such that the monopoly price of a successful innovator can be fully charged, resulting in a non-ambiguous replacement of the corresponding less competitive incumbent (see Aghion & Howitt (1992) for an analysis of the non-drastic case that yields similar comparative static results when the production function is of Cobb-Douglas type). The key issue is that each potential successful innovator can realize a strictly

[17] Given that our formulation of $G_{v,t}$ depends on technical levels $A_{k,t}$ achieved through learning-by-doing, Eq. (21) is strictly in line with the formulation of Jones (1995).

positive profit, yielding incentives to enter profit-motivated R&D activities. Turning now to the R&D technology, we follow Schaefer *et al.* (2014) and assume that the probability of success of a potential innovator in a specific sector k and machine line i writes

$$\lambda_{k,i,t} = \Phi_{k,i,t} H_{R,k,i,t} G_{v,t}, \tag{22}$$

where (i) $\Phi_{k,i,t,\kappa}$ captures the increasing difficulty to perform R&D with the current complexity of the corresponding production line $\kappa_{k,i,t}$, (ii) $H_{R,k,i,t}$ stands for the amount of human capital dedicated to research in the machine line i of sector k, and (iii) $G_{v,t}$ is the level of the current GPT vintage. It is worth mentioning that such a modeling choice of the probability of success does not preclude *per se* an upper bound for productivity gains in each sector. This is why, as advocated by Kortum (1993) and Stokey (1995), we explicitly introduce decreasing returns for the cost of R&D through the following functional form

$$\Phi_{k,i,t} = \frac{\bar{\lambda}_k - \lambda_{k,i,t}}{\phi_k} \frac{1}{q_k^{\kappa_{k,i,t}+1}}, \tag{23}$$

where $\phi_k > 0$ is a parameter capturing the cost of innovation in sector k, and $\bar{\lambda}_k \in (0,1)$ stands for the ultimate level of probability success in R&D. One can thus write the probability of an innovation success explicitly belonging to a bounded support as

$$\lambda_{k,i,t} = \bar{\lambda}_k \frac{H_{R,k,i,t} G_{v,t} q_k^{-[\kappa_{k,i,t}+1]}}{\phi_k + H_{R,k,i,t} G_{v,t} q_k^{-[\kappa_{k,i,t}+1]}}. \tag{24}$$

It is clear from Eq. (24) that the probability of successful R&D converges to its limit value, $\bar{\lambda}_k$, whenever the stock of human capital and the level of the GPT grow. Isolating $H_{R,k,i,t}$ from Eq. (24) allows to give a more intuitive interpretation for our specification:

$$H_{R,k,i,t} = \phi_k \times q_k^{\kappa_{k,i,t}+1} \times \frac{1}{G_{v,t}} \times \frac{\lambda_{k,i,t}}{\bar{\lambda}_k - \lambda_{k,i,t}}.$$

For each machine line $i \in (0,1)$ in sector $k \in \mathcal{K}$, the R&D effort, namely the level of human capital requirements to achieve a targeted probability sucess $\lambda_{k,i,t}$, decreases with the level of the current GPT, $G_{v,t}$, and increases with the fixed cost, ϕ_k, and the level of complexity of the corresponding machine line, $\kappa_{k,i,t}$, as well as the level of innovation success, $\lambda_{k,i,t}$. Moreover, due to the decreasing returns of R&D, the probability success becomes increasingly costly while approaching its upper bar $\bar{\lambda}_k \in (0,1)$.

Innovations decisions occur in machine line $i \in (0,1)$ of sector $k \in \mathcal{K}$, *prior* production decisions. They are driven by the monopoly profit that would reward a successful innovator, denoted by $\bar{\pi}^s_{k,i,t}$. R&D processes and monopoly profits being defined, we can then describe the behavior of innovator through the following free-entry condition.

Problem 5 (R&D – Sectoral innovators) *Each R&D sector $k \in \mathcal{K}$ is viewed as a pool of innovators, willing to enter the capital good production market through a successful innovation. Each potential monopolist takes as given the current level of GPT, $G(v,t)$, the complexity level of the targeted production line, $\kappa_{k,i,t}$, as well as the price of human capital, w_t, to maximize their expected profit, $\lambda_{k,i,t}\pi_{k,i,t}q_k$, such that at the equilibrium the following free entry condition in R&D holds*

$$\lambda_{k,i,t}\pi^s_{k,i,t} = w_t H_{R,k,i,t} \tag{25}$$

In writing this problem, we assume funds to be ultimately lent by households to potential innovators, and then repaid through profits (i.e. dividends) whenever innovation is successful. It is worth mentioning that market forces will endogenously shape the intensity of R&D in each sector. In other words, the monopoly profit that would reward a successful innovation depends on the technical advancement and the level of the energy resource specific to each sector. Thus, most promising sectors would tend to attract greater efforts to perform R&D, ultimately bounded by the labor force availability (market clearing condition defined shortly). In that sense, sectors that have greater potential concentrate more researchers/engineers, and this allocation is endogenous and might reverse. In addition, contrary to Acemoglu *et al.* (2012), this specification allows for innovation to potentially occur in both sectors simultaneously, which seems to be more suited to the analysis of long-term growth patterns. At last, we introduce the aggregate demand for human capital dedicated to R&D

$$H_{R,t} = \sum_{k \in \mathcal{K}} \int_0^1 H_{R,k,i,t} di.$$

3.4 Market-Clearing

At each time period, real flows must ensure that all markets – namely final good, physical capital, human capital, and financial assets – clear, that is

$$\begin{aligned} Y_t &= c_t N_t + (1+r_t) s_{t-1} N_{t-1} + K_{t+1} - (1-\delta) K_t + \sum_{k \in \mathcal{K}} \Psi_{k,t} E_{k,t} + b_t e_t N_t, && (26) \\ H_t &= \sum_{k \in \mathcal{K}} H_{k,t} + H_{R,t}, && (27) \\ K_t &= \sum_{k \in \mathcal{K}} K_{k,t}, && (28) \\ K_{t+1} &= s_t N_t. && (29) \end{aligned}$$

These conditions ensure that the provision of real flows equals their uses. It is worth mentioning that the constant returns to scale assumption for the production technologies, combined with the set of hypotheses conditioning the behavior of the household, ensure that Eq. (26) always holds at the equilibrium. Moreover, physical constraints shall hold in the provision of energy flows and in the provision of capital goods. Hence, we have

$$\begin{aligned} E_{k,t} &\leq R_{k,0} && \text{(renewable resource)}, \\ E_{k,t} &\leq R_{k,0} - \sum_{a<t} E_{k,a} && \text{(exhaustible resource)}, \\ K_{k,t} &= \int_0^1 x_{k,i,t} di, \forall k \in \mathcal{K}. && (30) \end{aligned}$$

4 Analytical Results

In this section, we provide some analytical results to build intuition about the key transition mechanisms from this paper.

4.1 General Equilibrium Solution

We turn now and for the rest of this paper to the general equilibrium solution of the model exposed in the previous section. More precisely, we look at the decentralized dynamic general equilibrium solution defined as follows.

Problem 6 (GE – General equilibrium) *An equilibrium is a sequence of: per capita consumption,* $\{c_t\}$*, savings,* $\{s_t\}$*, fertility* $\{b_t\}$*, educational investment* $\{e_t\}$*, physical capital allocations,* $\{\{K_{k,t}\}_{k \in \mathcal{K}}$*, human capital allocation,* $\{\{H_{k,t}\}_{k \in \mathcal{K}}$*, final input provision and primary energies' extractions flows,* $\{\{Y_{k,t}, E_{k,t}\}_{k \in \mathcal{K}}\}$*, as well as prices* $\{r_t, w_t, \{p_{k,t}\}_{k \in \mathcal{K}}, \{p^x_{k,t}\}_{k \in \mathcal{K}}\}$*, such that* *(i)* $\{c_t, s_t, b_t, e_t\}$ *solve Problem HH;*

(ii) $\{\{p_{k,t}\}_{k\in\mathcal{K}}\}$ *solve Problem FG;*

(iii) $\{\{x_{k,i,t}\}_{k\in\mathcal{K},i\in[0,1]}, \{H_{k,t}\}_{k\in\mathcal{K}}, \{E_{k,t}\}_{k\in\mathcal{K}}\}$ *solve Problem IP under primary energy resource constraints defined by Eq. (11) and (12);*

(iv) $\{\{p^x_{k,t}\}_{k\in\mathcal{K},i\in[0,1]}\}$ *solve Problem CG along with the free-entry condition of Problem R&D;*

(v) $\{r_t\}$ *and* $\{w_t\}$ *are such that the physical capital market, that is, Eq. (29), and the human capital market, that is, Eq. (27), clear;*

(vi) $\{\{x_{k,i,t}\}_{k\in\mathcal{K},i\in[0,1]}\}$ *are such that the capital resource constraint is satisfied, that is, Eq. (30) holds;*

(vii) Physical capital follows the accumulation dynamics described in Eq. (28);

(viii) Population and human capital follow the endogenous dynamics described in Eq. (6) and (8);

(ix) GPTs are generated from a non-homogeneous Poisson process of endogenous mean given by Eq. (14), and evolve according to Eq. (18) and (20);

(x) Learning-by-doing technical changes endogenously evolve according to Eq. (21);

(xi) R&D-based technical changes endogenously evolve according to Eq. (24) and (25).

As will be shown in the following subsections, at each time period, given the endowments inherited from the previous period (that is the amount of physical capital, technical levels and primary energy resources), one can solve for general equilibrium of the current period, ultimately setting the endowments for the next period. Thus, we adopt a dynamic recursive method to compute the general equilibrium just defined. We assume general equilibrium and provide now some analytical results about the theoretical model introduced in the previous section.

4.2 Preferences and Demography

At first, we focus on the households' behavior that defines the first transition mechanism in this paper: a demographic transition relying on a quantity-quality trade-off. To ensure a meaningful problem, that is, with a positive population size and non-negative education expenditures, we make the following assumption on preferences.

Assumption 3 *Preferences are such that,* $\eta > \rho$.

At each time period, given w_t and h_t (inherited from education during childhood), the first order conditions from Problem HH define the optimal allocation of time and income between present consumption, fertility, education expenditure and savings.

$$c_t = \frac{w_t h_t}{1+\chi+\eta}, \qquad s_t = \chi c_t, \qquad b_t = \frac{\eta c_t}{e_t + \tau w_t h_t}. \tag{31}$$

Provided that there is no minimum consumption constraint, adult households dedicate constant shares of their income in present consumption and savings. Moreover, it is clear from Eq. (31) that increasing education tends to reduce fertility. Turning now to child quality, there is an education threshold, $\tilde{Q}_t$, such that by monotonicity of $A_E(\cdot)$, we have

$$e_t = \begin{cases} 0 & \text{if} \quad Q_t < \tilde{Q}_t, \\ \frac{\left[\rho\tau A_E(Q_t)h_t - \eta\bar{h}\right]w_t}{A_E(Q_t)(\eta-\rho)} & \text{if} \quad Q_t \geq \tilde{Q}_t. \end{cases} \tag{32}$$

The threshold $\tilde{Q}_t$ is defined as the solution of $e_t(\tilde{Q}_t) = 0$, that is, $\tilde{Q}_t = A_E^{-1}\left(\eta\bar{h}/\rho\tau h\right)$. For a given level of human capital embodied during childhood, h_t, the schooling technology must be efficient enough for the adult planner to invest in it. It is interesting to denote that, by monotonicity and invertibility of $A_E(\cdot)$, we have $\partial\tilde{Q}_t/\partial h_t < 0$, which means that the per capita level of human capital tends to decrease the education threshold, $\tilde{Q}_t$. Thus, in this setting, education has a positive effect on the overall inclination of society toward schooling, more educated parents valuing more education. Precisely, we can state the following theorem.

Theorem 1 (Human capital accumulation) *Assuming that applied knowledge is initially insufficient for education expenditures to be strictly positive, that is $Q_t \leq \tilde{Q}_t$ initially, whenever the education threshold, $Q_t > \tilde{Q}_t$ at some period $t = T$, is met, the economy accumulates human capital, that is $\forall t \geq T, h_{t+1} > h_t$.*

Proof. See Appendix B.1. □

This theorem is key for the demographic transition that results from a quantity-quality trade off. More precisely, education expenditures, e_t, are initially null, because we assume that the stock of total knowledge, Q_t, is initially insufficient for the returns on education to be high enough to be worth investing in it. Hence, human capital is initially stuck at its minimal level, $\bar{h}$. At this point, the education threshold only depends on structural parameters of the model, that is $\tilde{Q}_t = A_E^{-1}(\eta/\rho\tau)$. Then, due to the technical improvements introduced in Section 3.3, the cumulative total stock of applied knowledge, Q_t, gradually increases up to the point where $\tilde{Q}_t$ is crossed. From then on, the representative adult starts to invest in education and human capital per capita begin to rise, initiating the demographic transition. More precisely, we can state the following theorem.

Theorem 2 (Quantity-quality trade-off) *The higher the desire for a large family, η (resp. for an educated family, ρ), the higher (resp. the lower) fertility, b_t, and the lower (resp. the higher) future levels of human capital, h_{t+1}. In addition, for an interior solution, the higher the wage rate, w_t, or the level of human capital, h_t, the lower fertility, b_t, and the higher education expenditures, e_t, and thus future levels of human capital, h_{t+1}.*

Proof. See Appendix B.2. □

Lastly, summarizing the previous results reveals two fertility paths. During the pre-modern regime, the representative household does not invest in education, human capital per capita is stuck at its lower value, $h_t = \bar{h}$, and fertility reaches an upper bound defined as

$$\bar{b} \equiv b_{h_t=\bar{h}} = \frac{\eta}{\tau(1+\chi+\eta)}.$$

In the modern growth regime, human capital per capita accumulates and is associated with a demographic transition. Fertility ultimately reaches a lower bound, defined as

$$\lim_{h_t\to\infty} b_t = \frac{\eta-\rho}{\tau(1+\chi+\eta)}.$$

4.3 Innovation and Production

We now turn to the endogenous supply of technologies and goods. Combining the first order conditions from Problem 3 and 4 allows to pin down the price of capital goods, as exposed in Appendix C.1. Whenever innovation is unsuccessful, the price of capital goods equals the marginal cost of production, that is $p^c_{k,i,t} = r_t + \delta$, which corresponds to the rental rate of raw capital, K_t, supplied by households. Whenever innovation is successful, the price of capital goods is $p^m_{k,i,t} = (r_t+\delta)/\alpha_k$, meaning that a mark-up $1/\alpha_k$ is applied by the monopolist on the competitive price. The resulting operational profit bears the incentive to perform R&D and is used to cover costs. It is worth noting that capital goods prices only depend on the competition regime and the sector, and not on the specific machine line. Then, we can show that the R&D success probability does not depend on the specific machine line either.

Proposition 1 (Sector-specific R&D success probability) *The R&D success probability only depends on the corresponding sector, $k \in \mathcal{K}$, and not on the specific machine lines, $i \in (0,1)$, that is*

$$\lambda_{k,t} = \bar{\lambda}_k - \frac{\phi_k w_t [p^m_{k,t}]^{\frac{\alpha_k}{1-\alpha_k}}}{G_{v,t}\bar{\pi}_{k,t}}. \tag{33}$$

Proof. See Appendix C.2. □

The expression in Eq. (33) is (i) increasing with the level of current GPT, $G(v,t)$, and with an indicator of the value of innovation, $\bar{\pi}_{k,t}$, and is (ii) decreasing with the sectoral research cost, ϕ_k, the wage level, w_t, as well as the cost of producing machines, $p^m_{k,t}$. As a result, human capital allocations, $H_{R,k,i,t}$, are uniformly distributed among all machines lines of a sector and do not depend on the quality ladder level, $\kappa_{k,i,t}$. In each sector, one can thus use Eqs. (22) and (23) to derive the aggregate amount of human capital dedicated to research as

$$H_{R,t} = \sum_{k\in\mathcal{K}} \frac{\phi_k}{G_{v,t}} \frac{\lambda_{k,t}}{\bar{\lambda}_k - \lambda_{k,t}} q_k Q_k. \tag{34}$$

In addition, the law of motion of the quality index, $Q_{k,t}$, can be computed in each sector by using the law of large numbers, ensuring that the probability of innovation success, $\lambda_{k,t}$, coincides with the fraction of machine-lines that will experience a success in R&D. This leads to the following quality dynamics

$$Q_{k,t+1} = \lambda_{k,t} q_k Q_{k,t} + (1-\lambda_{k,t})Q_{k,t}, \tag{35}$$

and thus the growth rate of innovation in each sector, $g_{Q_k,t}$, is

$$g_{Q_k,t} = [q_k - 1]\,\lambda_{k,t}. \tag{36}$$

Proposition 1 is also crucial for aggregation in each input sector. Indeed, as shown in Appendix C.3, one can compute the aggregate raw physical capital demand, $K_{k,t} = \int_0^1 x_{k,i,t}di$, and thus the aggregate production technology for each input k

$$Y_{k,t} = A_{k,t}[\tilde{Q}_{k,t}Q_{k,t}]^{1-\alpha_k} K^{\alpha_k}_{k,t} H^{\beta_k}_{k,t} E^{\gamma_k}_{k,t}. \tag{37}$$

Where $Q_{k,t} = \int_0^1 q_{k,i,t}di$ stands for an average quality index in sector k *after* innovation occurred at the beginning of period t. This quality index captures the technical level of sector k achieved through R&D, and thus shapes innovation and production decisions in period $t+1$. Whereas $\tilde{Q}^{1-\alpha_k}_{k,t}$ is a scaling parameter for the quality index that capture the hybrid nature of competition in the production of intermediate capital goods. This scaling parameter depending on R&D decisions (occuring *prior* to production) as the following expression,

$$\tilde{Q}^{1-\alpha_k}_{k,t} = \left[\alpha_k^{\frac{\alpha_k}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})\right] \Big/ \left[\alpha_k^{\frac{1}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})\right]^{\alpha_k}.$$

4.4 Equilibrium Results

In this section, we derive some theoretical properties from our model, focusing on a two-sector case that we will also consider for the numerical exercises of Section 5. The two sectors are respectively labeled *renewable* (indexed by r) and *exhaustible* (indexed by e), that is $\mathcal{K} = \{r, e\}$. In order to derive tractable analytical results, we hereafter consider that Assumptions 1 and 2 hold in addition with the following new assumptions.

First, we consider the particular case of symmetric technologies in final inputs production. This assumption ensures that the endogenous directed technical change only results from the endogenous changes in (i) energy input stocks, and (ii) their associated extraction technologies, and not from *ex-ante* biases with regard to production technologies.

Assumption 4 (Symmetry in production) *Production technologies of final inputs are assumed to be symmetric, that is* $\forall k \in \mathcal{K}, (\alpha_k, \beta_k, \gamma_k) = (\alpha, \beta, \gamma)$ *and* $\forall k \in \mathcal{K}, \vartheta_k = \vartheta$.

Second, to simplify the analytical resolution without altering the driving forces of our framework, we also assume for this section that extraction costs depend on current resource availability.

Assumption 5 *Extraction costs depend on current resource levels, that is* $\forall k \in \mathcal{K}, \Psi_{k,t} = \Psi(\mathcal{R}_{k,t}, Q_t)$.

Lastly, we assume final inputs to be gross substitutes to cope with historical facts and obvious physical properties of energy carriers.

Assumption 6 *Final inputs are gross substitutes, that is* $\sigma > 1$.

We may now state some results regarding equilibrium relative prices and quantities before turning to the endogenous direction of technical change that is the key transition mechanism discussed in this paper.

4.4.1 Equilibrium Prices and Quantities

In this section, we focus on relative prices and inputs quantities at equilibrium. We abstract from innovation decisions (extensively addressed after Section 4.4.2), provided they occur *prior* production decisions. From the optimality conditions of Problem 2, we obtain the isoelastic demand schedules for inputs, $Y_k = [p_k/\vartheta_k]^{-\sigma} Y$. Combining these expressions yields the usual substitution effect between CES inputs, that is, an inverse relationship between equilibrium relative prices and quantities,

$$\frac{p_{r,t}}{p_{e,t}} = \left[\frac{Y_{r,t}}{Y_{e,t}}\right]^{-\frac{1}{\sigma}}. \tag{38}$$

Substituting the equilibrium demands for inputs allows to derive a second expression for relative equilibrium input prices.

Proposition 2 (Relative Input Price) *Under Assumption 4, 5 and 6, the higher (resp. the lower) the relative level of learning and R&D knowledge (resp. the relative cost of extraction), the lower the relative price of final inputs.*

Proof. See Appendix D.1. □

At the equilibrium, relative prices react in a very intuitive way and tend to be higher in the sector having the greater extraction cost (cost-push effect) and lower in the sector technically more advanced (efficiency effect). Then, we can derive an opposite result with regards to the equilibrium relative demand for factors.

Proposition 3 (Relative Factor Use) *Under Assumption 4, 5 and 6, the higher (resp. the lower) the relative level of learning and R&D knowledge (resp. the relative cost of extraction), the higher the relative allocation of labor, physical capital, and energy resources.*

Proof. See Appendix D.2. □

Again, the interpretation of this property is standard, the sector that is technically more advanced or that exhibits a lower extraction cost are more efficient, and thus attracts relatively more production factors at the equilibrium.

4.4.2 Endogenous Direction of Technical Change

We focus now on the endogenous direction of technical change, which is the transition mechanism within our framework. It is obvious that whenever only one of the two sectors experiences R&D, a case that will appear in our numerical simulation, technical change is biased against the other sector. Here, we restrict to a situation where both sectors experience R&D to assess the dynamics of the underlying forces that shape technical biases. The endogenous direction of technical change can be assessed using the relative equilibrium growth rates of R&D knowledge that, according to Eq. (36), writes

$$\frac{g_{Q_r,t}}{g_{Q_e,t}} = \frac{q_r - 1}{q_e - 1}\frac{\lambda_{r,t}}{\lambda_{e,t}}. \tag{39}$$

Sector-specific quality improvements, q_k, and research success probabilities, λ_k, can be respectively interpreted as the sector-specific *magnitude* and the *scale* (i.e. mass of successful innovators) of R&D. Compared to the quite complex parametric form for the R&D success probability introduced in Eq. (24), we restrict in this section to a simpler formulation to derive tractable results. We adopt the following simplified functional form, which is usual in the Shumpeterian growth literature (e.g., see Aghion & Howitt, 2009)

$$\lambda_{k,i,t} = \bar{\lambda} G_{v,t}\left[\frac{H_{R,k,i,t}}{q_k^{\kappa_{k,i,t}+1}}\right]^{\epsilon}, \tag{40}$$

where $\bar{\lambda} \in (0,1)$ and $\epsilon \in (0,1)$ are parameters capturing the efficiency and the convexity of R&D efforts. Note that we assume these parameters to be identical for both sectors in the spirit of Assumption 4. The functional form of Eq. (40) is similar to the one introduced in Eq. (24), in the sense that the R&D success probability (i) increases with R&D efforts, $H_{R,k,i,t}$, (ii) exhibits decreasing marginal returns in R&D efforts, and (iii) decreases with the complexity of the machine line, $q_k^{\kappa_{k,i,t}}$. Moreover, combined with the free-entry condition in R&D in Eq. (25), the R&D success probability considered here also appears to be independent from the specific machine line as

$$\lambda_{k,t} = \bar{\lambda}^{\frac{1}{1-\epsilon}}\left[\frac{G_{v,t}\bar{\pi}_{k,t}}{w_t[p_{k,t}^m]^{\frac{\alpha}{1-\alpha}}}\right]^{\frac{\epsilon}{1-\epsilon}}. \tag{41}$$

The comparative statics of the equilibrium R&D success probability is also similar for both parametric forms: the higher the sector-specific profit perspectives, $\bar{\pi}_{k,t}$, and the level of the current GPT, $G_{v,t}$, (resp. the lower the production cost of machines, $p_{k,t}^m$, and the cost of R&D, w_t), the higher the R&D efforts and consequently the higher the sector-specific R&D success probability.[18]

[18] It can also be shown that the calibration of the two parametric forms, proposed respectively in Eq. (24) and (40), result in similar *behaviors*, in the sense of close numerical values for R&D success probability derived from the free-entry condition, i.e., from the underlying innovation incentives.

Substituting the previous equilibrium relations into innovator profits defined by Eq. (47) yields

$$\frac{g_{Q,r,t}}{g_{Q,e,t}} = \underbrace{\frac{q_r - 1}{q_e - 1}}_{\text{R\&D efficiency}} \times \underbrace{\left[\frac{p_{r,t}}{p_{e,t}}\right]^{\frac{\epsilon}{(1-\alpha)(1-\epsilon)}}}_{\text{price effect}} \times \underbrace{\left[\frac{H^{\beta}_{r,t}E^{\gamma}_{r,t}}{H^{\beta}_{e,t}E^{\gamma}_{r,t}}\right]^{\frac{\epsilon}{(\beta+\gamma)(1-\epsilon)}}}_{\text{market-size effect}} \times \underbrace{\left[\frac{A_{r,t}}{A_{e,t}}\right]^{\frac{\epsilon}{(1-\alpha)(1-\epsilon)}}}_{\text{productivity effect}}. \tag{42}$$

In a generalization of Acemoglu *et al.* (2012), the relative sectoral intensity in R&D then appears to be driven by: (i) an *R&D efficiency effect*, (ii) a *price effect* favoring innovation toward the sector with the higher price, that is the least advanced sector and/or the sector where the energy resource is the most scarce according to Proposition 2, (iii) a *market-size effect* favoring innovation in the larger sector in terms of employment and energy use (i.e., the larger potential market for complementary machines), that is, the most advanced sector and/or the sector where the resource is the most abundant according to Proposition 3, and (iv) a *direct productivity effect* favoring innovation in the more advanced sector in terms of learning-by-doing. It is worth mentioning that there is now a direct effect from the sector specific level of knowledge, $Q_{k,t}$, which is due to the specific parametric form of Eq. (40) where the normalization of R&D efforts by $q_k^{\kappa_{k,i,t}+1}$ exactly compensates for the effect from the machine-specific quality level in $\bar{\pi}_{k,t}$. However, as already seen in Proposition 2 and 3, relative R&D knowledge shapes equilibrium relative final input prices and factor use, indirectly featuring a *build-on-the-shoulders-of-giants* effect in R&D. To demonstrate this result, we consider an additional technical assumption regarding the competition regime for capital good provision.

Assumption 7 (CG production) *Capital goods are produced within a monopolistic competition regime.*

This assumption simplifies the analysis by ensuring that the scaling factor for the sector-specific R&D knowledge is constant, that is $\overline{Q}_{k,t} = \alpha^{\frac{\alpha}{1-\alpha}}$ and $\widehat{Q}_{k,t} = \alpha^{\frac{1}{1-\alpha}}$. However, it does not alter the fundamental drivers of innovation. Indeed, the hybrid competition regime presented in Subsection 3.3.3, where only machines' lines that experience successful R&D are supplied under monopolistic competition while others are supplied competitively, was introduced to ensure zero aggregate profits. Hence, labor is the only source of income for households, which result in bounded and tractable fertility choices. Relaxing this assumption would only magnifies the impact of the pre-existing R&D knowledge stocks $Q_{k,t}$.

Substituting the ratios of equilibrium relative prices and quantities within Eq. (42) allows the full characterization of the endogenous direction of technical change by

$$\frac{g_{Q,r,t}}{g_{Q,e,t}} = \frac{q_r - 1}{q_e - 1}\left[\frac{A_{r,t}}{A_{e,t}}\right]^{(\sigma-1)\frac{\epsilon}{1-\epsilon}} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{-\gamma(\sigma-1)\frac{\epsilon}{1-\epsilon}} \left[\frac{Q_{r,t}}{Q_{e,t}}\right]^{((1-\alpha)(\sigma-1)-1)\frac{\epsilon}{1-\epsilon}}. \tag{43}$$

Theorem 3 (Direction of technical change) *Under Assumption 4, 5, 6 and 7, and assuming that final inputs are sufficiently substitutable (i.e., that $(1-\alpha)(\sigma-1) \geq 1$), R&D technical change tends to be biased against the least advanced sector, both in terms of learning-by-doing and R&D knowledge, and/or the sector relaying on the energy resource that is the most scarce.*

Proof. A direct interpretation of Eq. (43) yields this result. □

This theorem illustrates the transition mechanism that is central within this paper. At first, the renewable sector is technically more advanced thanks to learning-by-doing ($A_{r,0} > A_{e,0}$), the corresponding extraction cost is slightly less expensive or similar ($\Psi_{r,0} \leq \Psi_{e,0}$), and R&D (if any) is more intense in this sector ($Q_{r,0} \geq Q_{e,0}$). Thus, the renewable resource is used at a larger scale than the exhaustible one, according to the equilibrium consumption ratio

$$\frac{E_{r,t}}{E_{e,t}} = \left[\frac{A_{r,t}}{A_{e,t}}\right]^{\sigma-1} \left[\frac{Q_{r,t}}{Q_{e,t}}\right]^{(1-\alpha)(\sigma-1)} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{-\gamma(\sigma-1)-1}.$$

However, as the renewable resource flow gets closer its ultimate maximum potential, its corresponding extraction cost, $\Psi_{r,t}$, increases. In the meantime the exhaustible extraction cost, $\Psi_{e,t}$, remains steady as the corresponding stock is initially nearly untapped. Moreover, due to the decreasing returns of learning-by-doing, the stock of knowledge in the renewable sector, $A_{r,t}$, ultimately grows less than in the exhaustible sector, $A_{e,t}$. These conflicting forces ultimately reverse the incentives to perform R&D, that becomes biased toward the exhaustible sector. Up until a certain point, and provided extraction costs do not reverse due to the depletion of the exhaustible resource, the direction of R&D toward the exhaustible sector become self-sustained (in the sense that a decrease in the $Q_{r,t}/Q_{e,t}$ ratio only triggers more R&D towards the exhaustible sector). This mechanism also triggers a reversal in the use of energy input, that is in the $E_{r,t}/E_{e,t}$ ratio, which materializes in an energy transition toward the exhaustible resource. In the next section, we illustrate this mechanism through numerical trajectories of a calibrated version of our model.

5 Numerical Analysis of the Transition Dynamics

In this section, we present the results of our numerical simulations. First, we focus on the historical experience of Great Britain for the period from 1700 to 1960. After introducing our baseline calibration, we discuss several counterfactual simulations to assess the role of coal in the timing and magnitude of the Industrial Revolution. We then turn to a comparison of "Western Europe" – i.e., the aggregation of Great Britain, Sweden, France, Germany, Italy, Spain, Portugal, and the Netherlands – and "Eastern Asia" – i.e., China–, two regions of the world that are roughly comparable in terms of territory, population, and resources during the last three centuries.[19] This exercise allows us to assess the comparative development analysis of Pomeranz (2000), who argues that the greater resource availability in Western Europe compared to Eastern Asia explains its earlier economic take of. More broadly, these exercises allow us to identify the deep rooted factors that triggered and sustained the transition from limited to sustained growth between these two regions of the world. Before analyzing these simulation results, we briefly describe the strategy employed to calibrate the model.

5.1 Calibration Strategy

Our numerical simulations begin in 1700 and follow a 20 years step. To ease the interpretation of the results, we normalize to unity in 1700 the relevant time-series that will be fitted (i.e. GDP, population, human capital per capita, total energy consumption).[20] In the simulations, we rely on Eq. (24) for the functional form for the probability of R&D success. This choice is motivated by the numerical tractability of this functional form, which is intrinsically bounded by $\bar{\lambda}_k$ and admits a bounded derivative whenever R&D efforts tend toward zero (e.g., $H_{R,k} \to 0$).[21] To account for the stochastic nature of endogenous GPT generation, we adopt a Monte Carlo approach and perform

[19]Pomeranz (2000, pp. 7–10) has extensively argued that what must be compared to Great Britain is not China as a whole, but only its most advanced development centre at the time of the Great Divergence, i.e. the Yangtze Delta. As a corollary, if the entirety of China is chosen for reasons of data availability, it must be compared to Western Europe.

[20]Consequently, we introduce a level parameter, Y_0, in the final good production technology, such that $Y_t = Y_0 \left[\sum_{k\in\{r,e\}} Y_{k,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ under Assumption 4 (notably $\forall k \in \mathcal{K}, \vartheta_k = \vartheta$). We consistently assume that initial stocks of learning-by-doing (e.g., A_k), R&D (e.g., Q_k), and general purpose (e.g., G_t) knowledge is normalized to unity. We also consistently assume that the level of human capital per capita absent education, $\bar{h}$, is normalized to unity.

[21]The alternative functional form, introduced in Eq. (40) to derive analytical results, would admit an infinite derivative, such that an equilibrium without R&D would not be possible.

10,000 runs for each of the scenarios presented below.[22] We graphically present truncated average and 90% $[0.05; 0.95]$ probability intervals of the obtained Monte Carlo trajectories.[23]

Due to the high dimensionality of the calibration problem, we rely at first on several assumptions. As for the theoretical analysis, we only consider two final input sectors respectively exploiting a *renewable* (indexed by r) and an *exhaustible* (indexed by e) energy resource. We chose the initial values of exhaustible energy and renewable stocks, $\mathcal{R}_{r,0}$ and $\mathcal{R}_{e,0}$, so that both would be half depleted by 2000 in the British and Western European cases; and in the case of Eastern Asia, we assume that the remaining exhaustible stock and renewable potential in 2000 would equal the Western European stock and potential for comparison purposes. To prevent any bias between final input sectors, we generalize Assumption 4 and impose a symmetric calibration for both sectors, except for extraction cost functions of primary energy and R&D productivity gains. More specifically, we assume that the scale, $\overline{\Psi}_k$, and the convexity, $\psi_{\mathcal{R},k}$, of the extraction cost functions may differ, and that one sector may generate greater quality improvements, q_k, than the other (which could however be the case provided coal is for instance more concentrated than wood). Thus, we do not expect one R&D technology to be cheaper or to have a higher frontier success probability. In what follows, we consequently drop the sectoral indexes k whenever there is no ambiguity due to this symmetry assumption. These assumptions ensure that, apart from endogenous dynamics, directed technical change and relative use of energy factors (i.e., the energy transition) will only result from clearly identified structural differences in the extraction cost functions of the two sectors.

In addition, we borrow the identifiable parameters related to production technologies from the literature. More precisely, α is set to its conventional $1/3$ value to match the share of capital (e.g., Acemoglu *et al.*, 2012), and γ to $1/6$, which is an intermediate value between the modern and preindustrial cost-share of energy in England (respectively approximately 5% and 25% according to Gentvilaite *et al.* (2015)). This leaves a share of labor, β, set to $1/2$. Due to our symmetry assumption for final inputs, we assume $\vartheta = 1$.[24] We also set the elasticity of substitution between final inputs, σ, to 4.4 as in Kander & Stern (2014). The usual assumption of a 5% annual depreciation rate for physical capital translates to setting δ to 0.64. We also assume that young adults save 20% of their revenue for an interior solution of Problem 1, that is $s_t/z_t = 0.2$, which allows us to express η as a function of χ according to Eq. (31).

At last, we use a best-fit calibration procedure to set the remaining parameters and initial values so as to replicate the historical times-series of energy resource flows, GDP, population, human capital per capita, and frequency of GPTs. We rely on sampling methods (design experiments protocols) to tackle the high dimensionality and computational intensity of our calibration problem. We basically us Latin Hypercube Sampling design with a *maximin* criteria[25] (Morris & Mitchell, 1995) to discriminate a wide range of calibration values using scoring metric based on squared-fit error with regards to historical time-series. The details of the calibration process are given in Appendix E.

5.2 The British Industrial Revolution

In our first numerical analysis, we focus on the British industrial revolution. Indeed, as presented in Section 2, some historian economists have emphasized the key role of coal as the main, if not sole, driver of the British early economic take-off (e.g. Allen, 2009; Pomeranz, 2000). To assess this

[22]We checked that the probability intervals do not significantly vary when increasing the number of simulations.

[23]We compute truncated averages, that is averages within the $[0.05; 0.95]$ probability interval of each simulated period, to rule out numerical errors that appear on a few runs at the upper tail of the probability distributions. In what follows, we refer to "truncated mean" whenever we mention "mean".

[24]To be precise, ϑ is set to 0.5 but the level parameter Y_0 that pre-multiplies the production technology in the final good sector allows to set the distribution parameters to 1.

[25]The LHS feature ensures good projective properties in each dimension of the calibration set, and the *maximin* criteria (maxmimizing the minimal distance across designs) ensures the diversity of the selected designs.

claim, we contrast a baseline calibration – set to reproduce historical data – to four counterfactuals. The latter are characterized by changes in the deep-rooted growth factors, namely energy extraction costs and resource levels, learning-by-doing accumulation – which is tightly linked to human capital accumulation as emphasized by Galor & Weil (2000) –, and GPTs diffusion speed. Our calibration database includes time-series about energy consumption (Kander *et al.*, 2013; Warde, 2007) [26], real GDP per capita (Bolt *et al.*, 2018; Broadberry *et al.*, 2015), population (Fouquet, 2014), human capital per capita (Lee & Lee, 2016), and GPTs' frequency (Lipsey *et al.*, 2005, p. 132). We show that once accounting for additional growth mechanisms, notably human capital accumulation, the energy transition towards coal appears much more as a catalyst than the root cause of the economic take-off during the Industrial Revolution.

5.2.1 Scenarios

We restrict our simulations to the 1700-1960 period to focus on a period in which the British energy consumption can reasonably be circumscribe to national resources (and does not depend too heavily on oil imports for instance). We show in the next case study how the model behave with a larger geographical area and longer time horizon. We distinguish the following scenarios:

- *Baseline* : follows the calibration procedure given above to fit the model to the British historical data. The obtained calibration set is given in Appendix F.
- *High cost*: is a counterfactual simulation where the level parameter, $\overline{\Psi}_e$, in the extraction cost function of the exhaustible sector is ten times as large as in the baseline scenario. This second scenario investigates the possible impact of a similar size but less concentrated exhaustible energy stock on the timing of the British industrial revolution.
- *Low diffusion*: is a counterfactual simulation where the diffusion speed of GPT, ξ, is ten times lower than in the baseline calibration. In this fourth scenario, we investigate the impact of knowledge diffusion on demography and its ultimate consequences on the transition from limited to sustained economic growth.
- *Low learning*: is a counterfactual simulation where the learning-by-doing productivity gains common to each final input sector, Ω, are ten times lower than in the baseline calibration. In this fifth scenario we assess the impacts on economic growth of lower efficiency of institutions in gathering and accumulating knowledge before the advent of formal R&D.

Consistently with the analysis from Section 4, we retrieve in the baseline calibration the following features: (i) for the same level of remaining resource and technology, it is relatively more costly to exploit the exhaustible resource, that is $\overline{\Psi}_r < \overline{\Psi}_e$ (e.g., cutting wood is *ceteris paribus* easier than mining for coal), (ii) the exhaustible extraction cost is relatively more convex with respect to the level of remaining stock, that is $\psi_{\mathcal{R},r} \leq \psi_{\mathcal{R},e}$ (e.g., mining for the last 10% of the remaining stock of coal is more expensive than to cut the analogous last 10% of the remaining wood resource), (iii) the overall extraction cost is higher in the exhaustible sector in the beginning of our simulations, that is $\Psi_{e,1700} > \Psi_{r,1700}$, and (iv) R&D quality improvements appear to be equivalent in both sectors, that is $q_r \approx q_e$ (e.g., no *ex ante* technical bias for a sector).

[26]The renewable primary energy resource of our model aggregates food, fodder, woodfuel, water and wind flows, whereas the exhaustible primary energy resource is an aggregate of coal, oil, gas, and nuclear.

5.2.2 Discussion: Coal as a Growth Catalyst

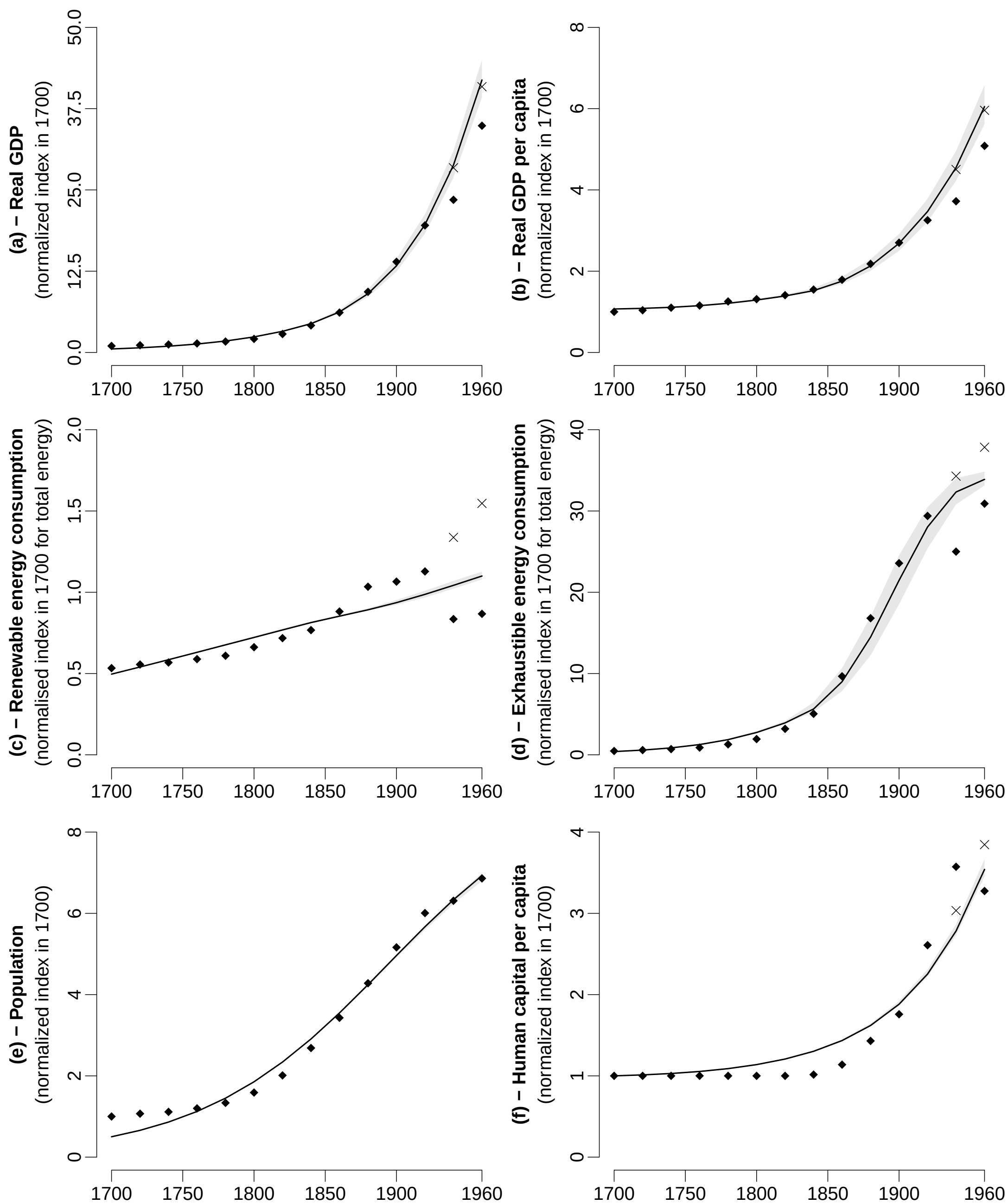
Figure 3 presents the numerical results of our *baseline* scenario, that is the mean and the $[0.05; 0.95]$ probability interval of the 10,000 runs, against the British historical data. The model replicates the time-series of GDP, population, GDP/capita, human capital per capita, and energy resource flows. The Great Depression of 1929 and World War II (WWII) induce a drop in most time-series in 1940 and 1960. The model is of course not designed to replicate such exceptional events and overshoots these historical points. For illustration purpose, we 'adjust' the 1940 and 1960 points with (exponential or quadratic) interpolations from 1900 to 2000, which are displayed in crosses. The fit of the *baseline* scenario is consistent with these adjusted data, including the GPT's frequency as displayed in Figure 9 of Appendix I, suggesting the model is a good representation of an economy absent major external shocks. One can notice that the model does not perfectly replicate the convexity of population time-series. The fit cannot be improved in our setting insofar as fertility exhibits a monotonous decreasing and bounded dynamics. However, introducing an additional survival consumption constraint within Problem HH, as in the seminal paper of Galor & Weil (2000), would solve this issue by generating an initial purely Malthusian ramp-up period for fertility. However, this refinement would significantly increase the computational intensity of the calibration procedure. Therefore, and the focus of this paper is to assess the impact of the energy transition on the economic take-off, we keep the population specification introduced so far.

We now turn to the counterfactual analysis displayed in Figure 4 and Table 1 to 4. Considering at first the *high cost* scenario, with regards to the baseline calibration, one can observe a 91% drop by in nonrenewable energy consumption while renewable energy resource consumption is 9% higher (substitution effect). As a result, the gradual transition towards exhaustible energy is delayed by almost a century. However, as the renewable resource flow reaches more rapidly its ultimate natural, it is not abundant enough to fuel economic growth. Accordingly, by 1900, this alternative energy transition dynamic results in an average 87% total energy consumption decrease and a 35% loss in GDP per capita in the *high cost* scenario compared to the *baseline* scenario. However, once technical change is advanced enough to compensate for the higher extraction cost of the exhaustible resource, the latter consumption significantly increases concomitantly with GDP and GDP per capita. Thus, a lower exhaustible resource quality only delays – and does not prevent – the industrial revolution by about 45 years. Moreover, it is worth mentioning that the detrimental impacts of an alternative *low stock* scenario[27] tend to be increasing and persistent, despite an initially lower magnitude (consistently with an initially lower extraction cost compared to the *high cost* scenario). The lack of available energy resources is indeed hindering economic growth, hence in 1960 a 35% decrease with regards to the *baseline* calibration in GDP per capita in such a *low stock* scenario, rather than a 17% decrease in the *high cost* scenario. However, one can still observe an acceleration of growth in the beginning of the XX$^{\text{th}}$ century sustained by human capital accumulation. As a result, once additional growth mechanisms such as human capital accumulation and technical changes are accounted for, our counterfactual simulations suggest that the energy transition towards coal was much more a catalyst than the root cause of the British economic take-off.

Next, the *low diffusion* scenario results in a lower consumption of both energy resources, which is more significant for the exhaustible one. Thus, in 1900, total energy supply is 31% lower than in the *baseline* calibration. In addition, the slower diffusion of GPTs implies a lower efficiency for both the learning-by-doing and the R&D processes. The cumulative nature of these processes generates a persisting and increasing gap. Moreover, the resulting hampered knowledge accumulation affects the quantity-quality trade-off, hence in 1960 an average 3.2% higher population level and an average

[27] Such a scenario would for instance consider a 10 times lower stock of exhaustible resource rather than a higher extraction cost (i.e., lower resource quality). The detailed results are available upon request.

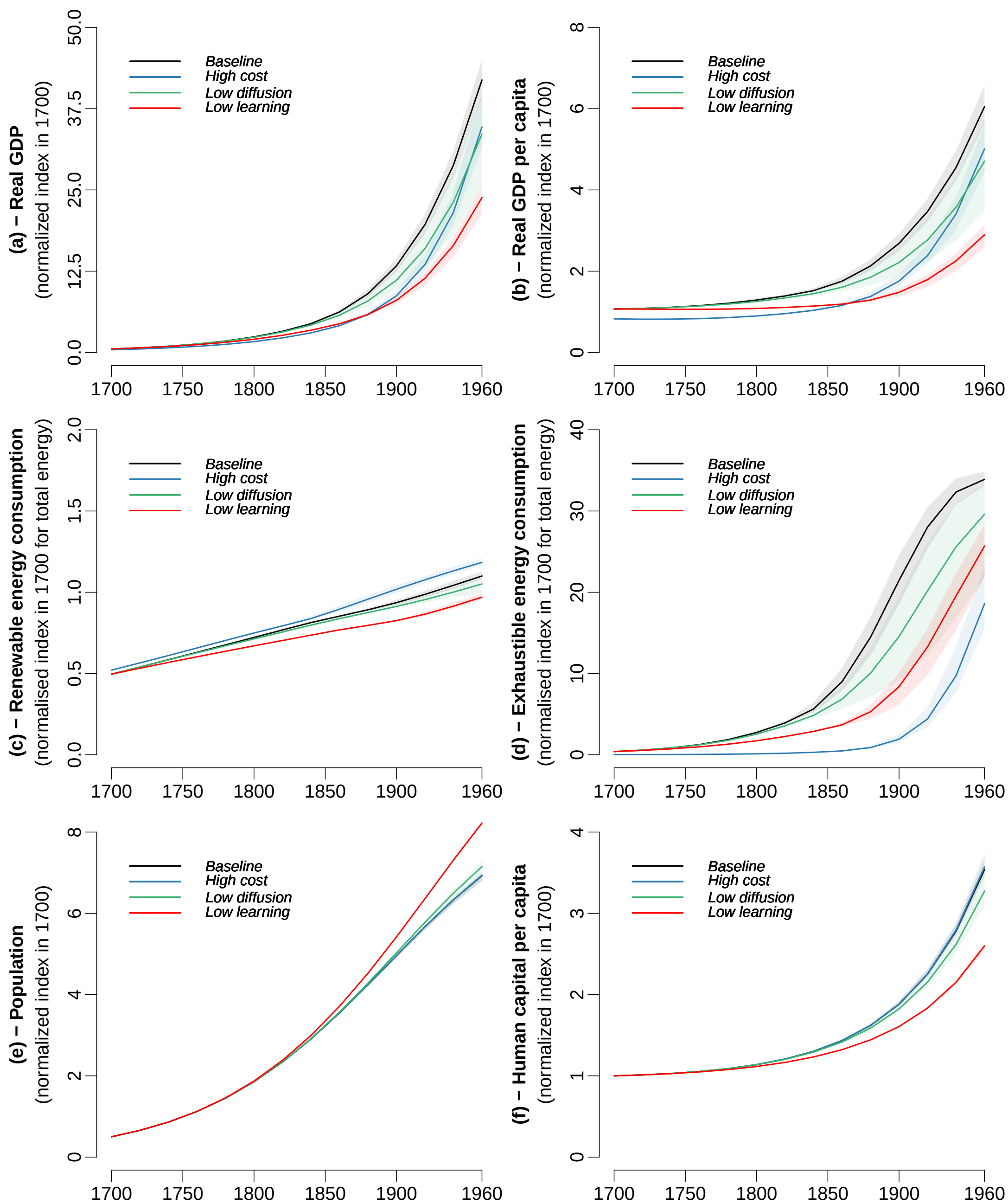
Fig. 3 Comparison of *baseline* calibration *(solid lines are averages, shaded areas represent 90% probability intervals)* with historical data *(diamonds)* and extreme-events-adjusted data *(crosses)*, 1700–1960.



7.5% loss in human capital per capita. The combined impact results in an average GDP per capita loss of 18% in 1900 and 22% in 1950 with regards to the *baseline* calibration.

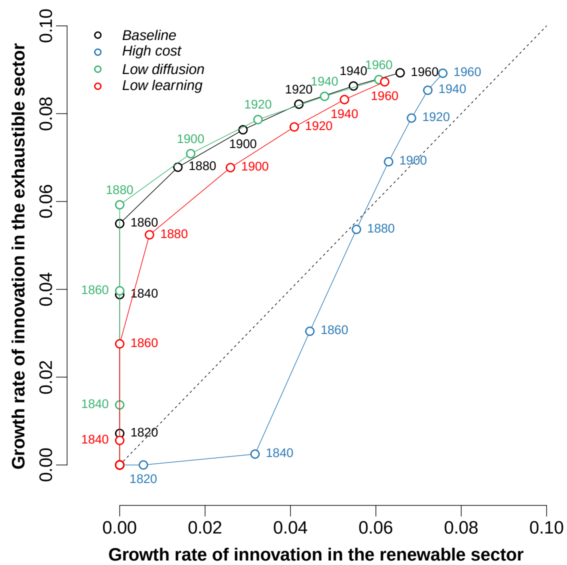
Turning to the *low learning* scenario emphasizes the crucial role of knowledge for demography, energy, and growth patterns. In this counterfactual, since learning-by-doing accumulate more slowly, the returns on education are initially not sufficient to make the investment worthwhile, hence a postponed children quantity-to-quality transition. The demographic transition is consequently postponed, which results in a larger population (+9% in 1900 and +19% in 1960) with an average lower human capital per capita (-17% in 1900 and -27% in 1960). The consequently lower supply in human capital penalizes energy extraction processes and delays R&D technical change, which

Fig. 4 Comparison of *baseline* calibration *(black)* with counterfactual simulations *high cost (blue)*, *low stock (green)*, and *low learning (red)*, 1700–1960 *(solid lines are the averages, shaded areas represent 90% probability intervals)*.



together generates a negative impact on economic growth. As a result, compared to the *baseline* calibration, the energy transition is delayed by about 40 years as both renewable and exhaustible energy consumption experience an average decrease of 12% and 61% respectively in 1900. Among all counterfactual simulations, the delayed energy transition of the *low learning* scenario generates the strongest lag in economic take-off of about 60 years, with relative average GDP per capita losses of 45% and 51% in 1900 and 1950 respectively. Moreover, the gap in GDP and GDP per capita is increasing over the period: despite a slight acceleration in the 1900-1950 period, there is no clear economic take-off in this scenario. This counterfactual suggests that, in line with the previous findings from Galor & Weil (2000), the interaction between technology and human capital through education is the key mechanism that drove the industrial revolution. As a corollary, the coal hypothesis does not seem to hold in a strong interpretation (i.e., as the sole cause of growth) once accounting for human capital, which was not the case in previous contributions to the literature (e.g. Otojanov, 2018; **?**). However, energy use is likely to have shaped the timing of the industrial revolution and its magnitude, acting as a catalyst of the previous mechanism.

Fig. 5 Phase diagram of the growth rate of R&D-based knowledge in the *baseline (black)*, *high cost (blue)*, *low stock (green)*, and *low learning (red)* calibrations *(points are averages, dashed line represents the first bisector)*.



We finally investigate more closely the direction of technical change toward the exhaustible resource sector, which is one of the central transition mechanisms explored in this paper. Figure 5 presents the phase diagram of the growth rate of R&D-based technical change in each sector, as given by Eq. (36). The area above (resp. below) the first bisector characterizes R&D technical change biased towards the exhaustible (resp. renewable) sector (that is a higher relative R&D growth). First, R&D-based technical change only starts in the XIX$^{\text{th}}$ century (1820 in the *baseline* calibration). Indeed, according to Eq. (33), R&D is initially not profitable enough – in terms of monopoly profits – due to insufficiently mature GPTs and efficient learning-by-doing technical change. Second, in all but the *high-cost* scenario, technical change is biased toward the exhaustible sector whenever R&D starts. This event coincides with the acceleration of both the exhaustible resource consumption and GDP growth. In particular, in the *baseline*, *low diffusion* and *low learning* cases, R&D even exclusively occurs in the exhaustible sector until 1860, 1880, and 1860 respectively. This is in line with our analytical results: rising extraction costs in the renewable sector, due to a shortage in the available resource – in 1800, about 50% of the stock of the renewable resource is extracted in the *baseline* scenario against a mere 2% of the exhaustible one –, (re)directs R&D-based technical change towards the exhaustible sector. The *high cost* scenario also illustrates this mechanism. A higher extraction cost in the exhaustible sector initially drives innovation toward the renewable sector. However, as 63% of the renewable resource is extracted by 1880, the associated rising extraction cost reverses previous R&D incentives towards the exhaustible sector, which coincides with a delay in the acceleration of economic growth. Third, at the turn of the XX$^{\text{th}}$ century, R&D in the renewable sector gains momentum in the *baseline*, *low diffusion*, and *low learning* scenarios, resulting in a more balanced but still biased technical change at the end of the simulation horizon (1960). Indeed, due to knowledge accumulation, more frequent GPTs arrival and diffusion, and rising costs in the exhaustible sector (about 30% of the exhaustible stock has been extracted by 1940 in the *baseline scenario)*), R&D also becomes competitive in the renewable sector, thus reducing (but not offsetting) the initial technical bias towards the exhaustible sector.

Table 1 Percentage change in renewable (E_r), exhaustible (E_d), and total ($E = E_r + E_e$) energy consumption, between the average trajectories of counterfactual scenarios and the *baseline* calibration.

Chg. w.r.t. Baseline	ΔE_r in % 1800	1900	1960	ΔE_e in % 1800	1900	1960	ΔE in % 1800	1900	1960
High cost	+3.8	+8.8	+7.6	-95.7	-91.1	-45.2	-75.0	-86.9	-43.5
Low diffusion	-1.2	-2.5	-4.4	-7.4	-32.2	-12.6	-6.1	-31.0	-12.4
Low learning	-7.1	-11.8	-11.8	-37.1	-61.0	-24.2	-30.8	-58.9	-23.8

Note: For any variable $X \in \{E_r, E_e, E\}$ of any counterfactual scenario (c), we denote the percentage change with respect to (w.r.t.) the baseline (b) as ΔX_c, with $\Delta X_c = (X_c - X_b)/X_b$.

Table 2 Percentage change in GDP (Y) and GDP per capita (Y/P), between the median trajectories of counterfactual scenarios and the *baseline* calibration.

Chg. w.r.t. Baseline	ΔY in % 1800	1850	1900	1950	$\Delta(Y/P)$ in % 1800	1850	1900	1950
High cost	-30.4	-32.9	-34.6	-21.3	-30.4	-32.9	-34.6	-21.2
Low diffusion	-2.6	-6.3	-16.5	-19.8	-2.7	-6.7	-17.5	-21.8
Low learning	-15.2	-26.0	-39.8	-43.0	-16.1	-28.7	-44.9	-51.3

Note: For any variable $X \in \{Y, Y/P\}$ of any counterfactual scenario (c), we denote the percentage change with respect to (w.r.t.) the baseline (b) as ΔX_c, with $\Delta X_c = (X_c - X_b)/X_b$. Given that the time step of the model is 20 years, starting in 1700, the results for 1850 and 1950 are obtained through exponential interpolations of our simulated results.

Table 3 Percentage change in the compound annual growth rate of GDP ($CAGR_{Y/P}$) and exhaustible energy ($CAGR_E$) between the average trajectories of counterfactual scenarios and the *baseline* calibration.

Chg. w.r.t. Baseline	$CAGR_{Y/P}$ in p.p. 1800-1850	1850-1900	1900-1950	$CAGR_E$ in p.p. 1800-1850	1850-1900	1900-1950
High cost	-0.08	-0.05	+0.38	-0.94	-0.37	+2.42
Low diffusion	-0.08	-0.23	-0.08	-0.26	-0.37	+0.39
Low learning	-0.28	-0.42	-0.11	-0.64	-0.41	1.03

Note: For any variable $X \in \{Y, Y/P\}$ of any counterfactual scenario (c), we denote the percentage change with respect to (w.r.t.) the baseline (b) as ΔX_c, with $\Delta X_c = (X_c - X_b)/X_b$. Given that the time step of the model is 20 years, starting in 1700, the results for 1850 and 1950 are obtained through exponential interpolations of our simulated results.

5.3 Comparative Analysis of Western Europe and Eastern Asia

In our second numerical exercise, we turn to a comparative analysis of Western Europe (i.e., the aggregation of Great Britain, Sweden, France, Germany, Italy, Spain, Portugal, and the Netherlands) against Eastern Asia (i.e., China), hereafter respectively labelled WE and EA. We aim to assess whether energy resource quality (i.e., relative extraction costs) have shaped development patterns, as advocated by Pomeranz (2000). We thus compare a baseline calibration for each geographical area – set to replicate historical data –, and a counterfactual scenario for EA. Our calibration database includes time-series about energy consumption (Kander *et al.*, 2013), real GDP per capita (Bolt *et al.*, 2018; Broadberry *et al.*, 2018; Fouquet & Broadberry, 2015; Malanima, 2011), population (Kander *et al.*, 2013; Maddison, 2007; Malanima, 2009; World Bank, 2018), and human capital per capita (Lee

Table 4 Delay (years) to reach the average levels of GDP per capita ($Delay_y$) and total ($E = E_r + E_e$) energy consumption ($Delay_E$) of the *baseline* scenario for the counterfactual scenarios.

Chg. w.r.t. Baseline	$Delay_y$ in yr. 1800	1850	1900	$Delay_E$ in yr. 1800	1850	1900
High cost	+72	+44	+26	+104	+79	n.r.
Low diffusion	+8	+13	+17	+4	+12	+25
Low learning	+80	+60	+53	+36	+42	+46

Note: "n.r." means "not reached within the simulated horizon (1700-1960)". The top-left square reads "in the *high-cost* scenario, it take 23.1 more years to reach the GDP level of the *baseline* scenario in 1850 ". Given that the time step of the model is 20 years, starting in 1700, the results for 1850 and 1950 are obtained through exponential interpolations of our simulated results.

& Lee, 2016).[28] Our counterfactual analysis reveals that energy resource quality help to explain the timing differential in the economic take-off in these two world regions, which again supports the coal-hypothesis in a weak sense.

5.3.1 Scenarios

We adopt a 1700-2000 simulation period to encompass the take-off periods of both WE and EA. We distinguish the following scenarios:

- *Baseline WE* : follows the calibration procedure given in Section 5.1 to fit the model to WE Western European historical data. The obtained calibration set is given in Appendix G.
- *Baseline EA* : follows the general calibration procedure given in Section 5.1 to fit the model to EA historical data. The obtained calibration set is given in Appendix H.
- *Alternative EA* : is a counterfactual simulation for Eastern Asia in which we set the values of the main pre-industrial deep-rooted growth factors to an order of magnitude similar to their WE counterparts. More precisely, (i) the renewable resource extraction cost parameters, $\overline{\Psi}_r$ and $\psi_{\mathcal{R},r}$, are multiplied by 3 and 1.5 respectively, (ii) the learning-by-doing technical change, Ω, is multiplied by 10, and (iii) the general productivity, Y_0, is multiplied by 1.3. The other parameters and initial values, such as the initial levels of energy resources, $\mathcal{R}_{k,0}$, are maintained to their *baseline EA* levels.

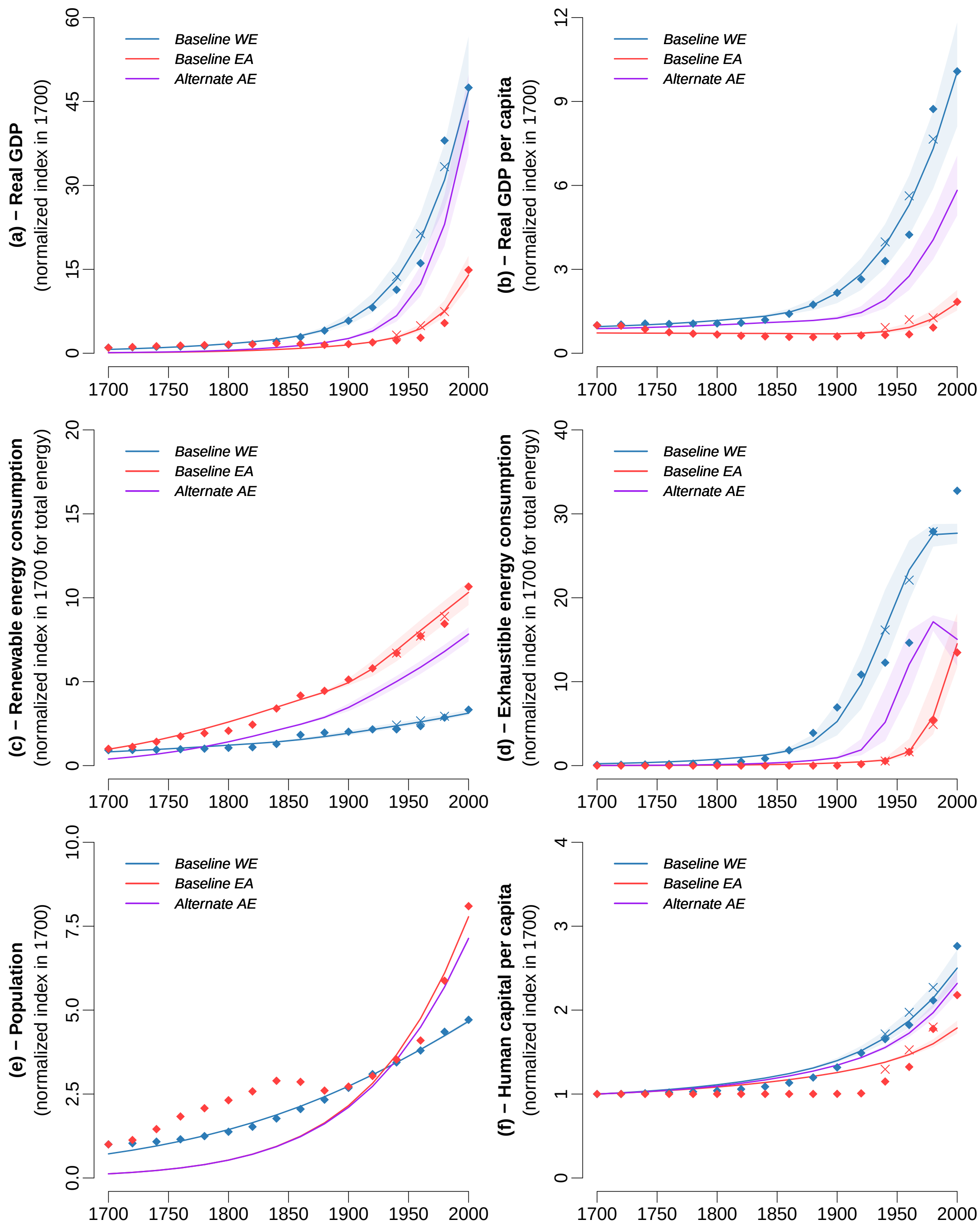
5.3.2 Discussion: Energy Quality as a Meaningful Mechanism for Comparative Development Analysis

Figure 6 presents the numerical results of our three scenarios, that is the mean and the $[0.05; 0.95]$ probability interval of the 10,000 runs, against historical data. The overall fit is satisfactory for both *baseline WE* and *baseline EA*, except for the later population and human capital time-series. As already mentioned, this is a drawback from the monotonic fertility pattern arising in our framework, while the Eastern Asian case clearly exhibits a ramp-up period for fertility. Again, introducing a survival consumption threshold as in Galor & Weil (2000) would solve this issue but make the calibration process more complex, which we refrain from doing in this paper. Moreover, in both *baseline WE* and *EA* calibrations, the model tends to over-fit historical data during the extreme events of WWI and WWII, obviously not modelled within our framework. The resulting additional depletion of the exhaustible resource with regards to historical data, coupled with a strengthening globalization since the 1970s – associated with increasingly intense international oil flows –, might explain

[28]Exponential or polynomial interpolations were performed to reconstruct missing occurrences in the time-series that were not given at a year-by-year basis.

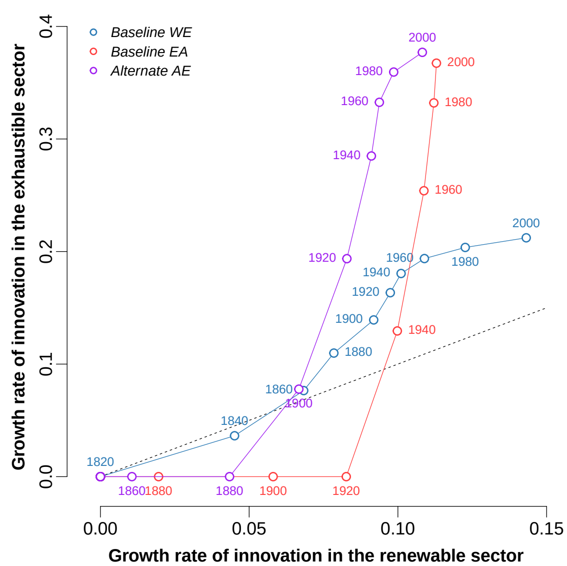
why the model tends to then under-fit the historical exhaustible energy consumption in the last period of simulation. For illustration purposes, we again 'adjust' the 1940, 1960 and 1980 points with interpolations from 1900 to 2000, which are displayed with crosses. The fit of the two *baseline* scenarios is consistent with these adjusted data points, which suggests the model is again a good representation of an economy in the absence of major external shocks.

Fig. 6 Comparison of *baseline WE* calibration *(blue)* with *baseline EA* calibration *(red)* and *alternative EA* calibration *(purple)*, 1700–2000, *(solid lines are averages, shaded areas are 90% probability intervals, diamonds are historical data and crosses are extreme-event-adjusted data).*



A comparison between the *baselines* calibration reveals three main pre-industrial differences between the two world regions.[29] First, renewable energy appears to be relatively easier to access (i.e., relatively lower extraction cost, that is relatively better quality) in EA than in WE. Second, the efficiency of learning-by-doing technical change is about one order of magnitude lower in EA compared to WE (-93% difference in the Ω parameter).Third, this significant difference in technical efficiency is magnified by the initial general productivity, Y_0, that is about 30% higher in WE compared to EA. When parameters are changed accordingly in the *alternative EA* scenario, the pattern of the simulation becomes much more similar to the *baseline WE* scenario. This is mostly visible in terms of the timing of the real GDP take-off and trajectories of exhaustible energy consumption flows, and confirmed in the pattern of R&D-bias as displayed in Figure 7. This phase diagram shows that in the *baseline EA* scenario, only the renewable sector experiences significant R&D until the end of the XIX$^{\text{th}}$ century. Thus, in line with the above-mentioned lower accumulation of learning-by-doing technical change and more concentrated energy in the renewable resource, R&D is initially strongly biased against the exhaustible sector. It only become profitable enough to initiate by the middle of the XX$^{\text{th}}$ century, concomitantly to the observed economic take-off. By contrast, in the *alternative EA* scenario, directed technical change is more rapidly biased toward the exhaustible sector, notably because knowledge accumulation is more efficient – which also implies that GPTs become more frequent and diffuse more widely – and because the renewable resource is more costly. Again, the shift towards a strong exhaustible sector R&D-bias in both the *baseline WE* and *alternative EA* scenarios coincides with the large-scale use of the exhaustible resource and the beginning of the economic take-off. Thus, resources quality endowments discrepancies are relevant to explain the comparative dynamics of Western Europe and Eastern Asia, as suggested by Pomeranz (2000). However, it is worth highlighting again that once accounting for the interaction between technologies and human capital, as emphasized by Galor (2005), energy use does not appear any more as the root-cause of the economic take-off.

Fig. 7 Phase diagram of the growth rate of R&D-based knowledge in the *baseline WE (blue)*, *baseline EA (red)* and *alternative EA (purple)* calibrations *(points are averages, dashed line represents the first bisector)*.



[29]This comparison is relevant provided historical time-series have been normalized to unity in the first simulation period (i.e., 1700), and the general calibration procedure was applied separately for WE and EA.

6 Conclusion

In this article, we assess the role of energy in long-term growth also accounting for more conventional transition mechanisms extensively documented in the literature, namely the accumulation of human capital and its interaction with technologies. To do so, we develop an endogenous growth model of a closed economy featuring fertility choices, energy extraction and knowledge accumulation. We analytically show that a shortage in renewable energy, such as wood, can be effective at triggering biased technical change towards a more abundant resource, such as coal. We then take the model to the data and investigate the transition from limited to sustained economic growth for two historical episodes: the British Industrial Revolution and the comparative dynamics of Western Europe and Eastern Asia.

Regarding the British case, our counterfactual analyses suggest that the energy transition towards coal in the XIX$^{\text{th}}$ century does not appear as the root-cause of the economic take-off, but rather acts as a catalyst required to observe contemporary levels of economic developments. Our numerical simulations indeed suggest that raising the extraction cost of coal by one order of magnitude – that is degrading fossil energy accessibility – delays the industrial revolution by about 45 years. Moreover, the interaction between technology and human capital accumulation is central in unlocking the industrial potential of the exhaustible energy resource and triggering the demographic transition. Lowering learning-by-doing technical change by one order of magnitude has the consequence of dividing the exhaustible energy consumption by a factor of more than two in 1900, resulting in a 45% drop in output per capita. In this case, the resulting delayed demographic transition magnifies the downside impact of knowledge accumulation, hindering human capital accumulation, R&D technical change, and exhaustible energy extraction. Regarding the comparative development of Western Europe and Eastern Asia, our numerical analyses suggest that discrepancies in fossil energy accessibility and learning-by-doing technical change, relatively higher in Western Europe, can explain the observed timing and magnitude differentials in the economic take-off.

The analysis of the knowledge-energy-demography nexus of this article thus sheds some light on both the timing and the magnitude of the industrial revolution. It thus supports a coal hypothesis of economic growth in a weak sens, meaning that energy resource endowments cannot by themselves account for the timing and magnitude of economic development trajectories and need to be analysed in interaction with human capital and technical change. However, these findings call for a better consideration of energy in the analysis of future long-term growth patterns due to its significant role as a catalyst, in particular regarding the energy shift towards modern renewable technologies that is needed to tackle global climate change.

Appendix A Detailed Evidences on Drivers of Economic Development

A.1 The controversial child quantity-quality trade-off

If some studies, such as Cáceres-Delphiano (2006) for the USA, Li *et al.* (2008) for China, and Becker *et al.* (2010) for Prussia, find the expected negative family-size/child-quality relationship, other empirical studies, such as Angrist *et al.* (2010) for Israel, Black *et al.* (2005) for Norway, and Clark & Cummins (2016) for England, find no evidence of such a quantity-quality trade-off. Regarding the emblematic case of Britain on the period 1780–1880, Clark & Cummins (2016) find that family size did not affect education, occupation, longevity, or even wealth. On the wider 1580–1830 period, Wrigley *et al.* (1997, p. 461) suggest that natural fertility was the norm in England, so that small groups may have been practising family limitation, but the reconstitution evidence suggests that such behavior was restricted to a small minority of the population, if present at all.

Clark & Cummins (2016) conclude that modern growth consequently cannot be explained by a switch to smaller family sizes accompanied by more investment in child quality. Modern growth in England had begun 100 years before there were significant reductions in average family sizes. This one hundred year delay between the ignition of the accelerated economic growth and the onset of the demographic transition in England suggests that alternative mechanisms should be called on to better relate the initiation of the transition towards sustained economic growth to the apparently subsequent (and not simultaneous) demographic transition. However, a more recent study performed by Klemp & Weisdorf (2018) finds that parental fecundity positively affected the number of siblings and that children of parents with lower fecundity were more likely to become literate and employed in skilled and high-income professions. In summary, there is no systematic evidence that the child quantity-quality trade-off exist.

Moreover, there is also no consensus on the idea that the quantity-quality trade-off, if it exists, is the main driver of the demographic transition. Different social scientists have suggested that social norms, the large declines in mortality starting in the nineteenth century, and the reduced need for child labor are potential factors contributing to the demographic transition. Becker (1981) was the first to formalize a theory relating the quantity-quality trade-off of households to the rise in demand for human capital. But twenty years before, Becker (1960) advanced the much simpler argument that the decline in fertility was a by-product of the increase in income and the associated rise in the opportunity cost of raising children. This theory hinges on the supposition that individual preferences reflect an innate bias against child quantity beyond a certain level of income. This mechanism was recently modeled by Strulik *et al.* (2013). Before them, Jones (2001) used a simplified version of the same approach with a formal representation of the mortality rate, which allowed him to reproduce the fact that mortality rates decrease before fertility rates in countries experiencing a demographic transition.

A.2 The exaggerated role of human capital for take-off

One can cast some doubts on the central role that unified growth models accord to human capital of the general population in fostering economic take-off. Indeed, Mokyr (2011, p. 232), notices the weak accomplishment of schooling to build human capital that would be useful to reach a modern regime. According to him, even in the eighteenth-century, public education in Britain was primarily destined to educate gentlemen in the traditional sense of the word, that is, men without a well-defined occupation whose curricula consisted of the classics, languages, and other humanities. Besides, Mokyr (2011, p. 239) shows that adult literacy rates in Britain *circa* 1800 were equivalent to those of France and Belgium, and were even lower than those of the Netherlands. Moreover, Mokyr (2011, p. 239) asserts that even if Britain rapidly became richer than other countries thanks to its early economic take-off, its ability or willingness to educate its young did not appreciably improve during the first phase of the Industrial Revolution. At the end of the nineteenth century, school enrollment was indeed not higher in Britain relative to countries that experienced delayed takeoffs such as Prussia or France.

Finally, as an unequivocal criticism of the crucial role that most unified growth models assign to general human capital, Mokyr (2011, p. 240) adds that, at the time of the British economic take-off, human capital was surely not the result of an investment process in which the human capital rate of return on the margin would be equal to the interest rate. Rather, for Mokyr, it might well be that the causal direction was reversed and that many people decided for non-economic reasons to educate their children and then discovered that this education imparted economically useful capabilities. He then concludes that in any event, to the extent that the available data allow to make inferences, the notion that the Industrial Revolution depended a great deal on human capital as customarily defined is not sustained. On the contrary, Mokyr (2011, p. 233) asserts that the great English engineers of the Industrial Revolution learned their skills by being apprenticed to able masters, and otherwise were largely self-taught. The latter observation suggests that learning-by-doing used to play a prominent role in the pre-modern growth regime.

A.3 The misguided reasons for the omission of energy

Assigning a modest importance to energy in explaining growth is conventionally justified by its small share in national income. Indeed, the so-called 'cost share theorem' implies that, if the aggregate production function is homogeneous of degree one, the output elasticities of production factors equal their income allocation in total GDP. Consequently, GDP elasticities with respect to labor and capital are generally set to 0.7 and 0.3 according to their respective empirical shares of GDP, while energy is usually neglected because its cost usually represents around 5% of the national income. Even when it is considered as a production factor, the output elasticity of energy is set to 0.05, such that labor and capital remain the most important production factors (Denison, 1979). However, it can be argued that this 'cost share theorem' is fallacious for several reasons.

First of all, the cost share theorem results from a Lagrange optimization assuming that all perfectly competitive markets are at equilibrium for an economy only composed of small price-taking firms. Consequently, the cost share theorem is only true at the margin (for a fictive economy), such that output elasticities with respect to a given input follow the income cost share of those inputs only for small shocks. Moreover, by construction, GDP is allocated exclusively to capital and labor payments. Accordingly, energy expenditure is itself only made of capital and labor payments (plus temporary market powers).[30] But the fact that energy expenditures are relatively low in developed economies does not imply that energy *per se* is of no importance for economic growth. This fact was well illustrated by the first energy crisis of 1973, during which a 5% decrease in oil availability induced a 3% loss of GDP in the US, which is much higher than the mere 0.25% that the cost-share theorem predicted. In a review on how energy price shocks affect the US economy, Kilian (2008) asserts that rising energy prices cause both a reduction in aggregate demand, and a shift in consumer expenditures, which in turn create a ripple effect throughout the economy. The effects of energy price shocks on economic output are hence larger than the share of energy in income would suggest. This means, that the output elasticity of energy of 0.05 generally presupposed in standard macroeconomics is underestimated, whereas the output elasticities of capital and labor of 0.3 and 0.7 respectively, are overestimated.

Furthermore, energy expenditures used to account for up to 70% of national income in pre-industrial, low-growth economies It is likely only the use of previously untapped concentrated, and consequently cheap, fossil fuels that this value gradually declined below 10% (Fizaine & Court, 2016). Kander *et al.* (2013, p. 7) assert that the decrease in the cost of energy, concurrent with an increase in supply, allowed vast reserves of capital to be employed, delivering other kinds of goods and services rather than covering only basic energetic needs as was the case during pre-modern times. Hence, the small cost share of energy in modern economies is not a sign of its worthlessness, but on the contrary, it might indicate the crucial importance that concentrated fossil energy has on modern economic growth.

Finally, the ground-breaking work of Kümmel & Lindenberger (2014) shows that, whenever *hard* technical constraints – corresponding to 'limits to automation' and 'limits to capacity utilization' – are taken into account, shadow prices add up to usual factor costs, implying that the cost share theorem simply no longer holds.[31] In summary, pure financial expenditure accounting downplays the role of energy because it does not take into account the interrelation between energy and specific technical developments that have been crucial to generating an expansion of many sectors of the economy. For instance, the design of modern transport systems and the associated suburban habitat

[30] For instance, the price of gasoline is constituted of capital interest, labor payments, and various taxes that are required to extract and refine the crude oil provided free-of-charge by nature.

[31] Besides, Ayres *et al.* (2013) argue that there are also some *soft* constraints – corresponding to social, financial, organizational, or legal restrictions – that determine additional limits to substitution between inputs over time.

have been wholly dependent on the Internal Combustion Engine (ICE) fueled by gasoline (and similarly, electric or gas-fired heating and cooling systems have made domestic and office life bearable in a variety of climates).

A.4 Distinguishing several 'kinds' of energy

In order to understand the importance of energy for the economic process, it is crucial to distinguish between primary, final, and useful energy.[32]

Primary energy is present on earth in the form of natural stocks (coal, oil, gas, and fissil atoms such as uranium) or flows (from the sun, water, wind, geothermal, waves and tides) that must be converted into secondary energy carriers in order to be usable. Such final energy vectors consist of heat flows, electricity, and solid, liquid or gaseous refined products. Then, end-use devices allow the conversion of final carriers into useful energy in the form of motion (i.e., mechanical drive), electricity, heat, and light. Energy services (transportation, heating, etc.) are the outcomes of the interaction of useful energies with capital and labor.

Because technical change impacts each conversion step of energy systems – from primary to final to useful stages – with different magnitudes, the prices of primary, final, and useful energies do not evolve similarly. An example of such difference is given in Figure 8, where the average price of primary energy is compared to the average price of energy services in Great Britain from 1700 to 2000. As Fouquet (2011) argues, focusing on the price of primary energy rather than the price of energy services (or useful energy) can lead to flawed reasoning because the former ignore major technical improvements developed to provide the latter.

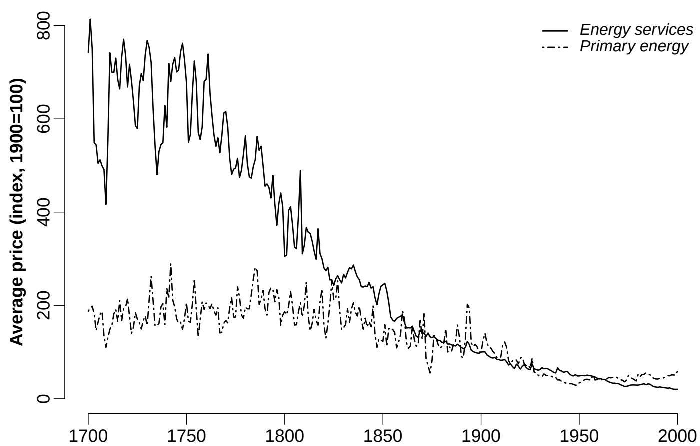


Fig. 8 Average prices of primary energy *(dashed line)* and energy services *(solid line)* in the United Kingdom, 1700–2008 *Data source*: Fouquet (2011).

In this paper, we accordingly distinguish: (i) primary energy exhaustible stocks and renewable potentials (i.e., flows), (ii) energy carriers that enter production processes and are combined with

[32] As repeatedly stressed by scholars such as Ayres & Warr (2009) and Kümmel (2011), what is commonly called energy in economic studies and models is in fact *exergy*. Exergy (measured in joules similarly to energy) is the maximum amount of work – in the mechanical sense – that can theoretically be recovered from a system as it approaches equilibrium with its surroundings reversibly, that is, infinitely slowly. According to the first law of thermodynamics, energy is conserved in the economic process; while, according to the second law of thermodynamics, exergy is dissipated through irreversible transformations that imply entropy creation. Energy enters the economy as a high quality (high exergy content) input in the forms of concentrated solar energy (biomass and water/wind flows), geothermal and tidal potential, fossils fuels, and nuclear energy. These energy forms are ultimately dissipated into a lower-quality (lower exergy content) heat output that potentially contains zero exergy (and thus zero ability to generate useful work) if its temperature is the same as the broader environment. Hence, it is the exergy content of energy that constitutes a production factor and not energy *per se*. In this article we stick to the familiar term of energy, even if, strictly speaking, we refer to exergy.

capital, labor, and knowledge, and (iii) energy services ultimately combined in final goods' production. We thus account for the technical improvement, and increases in non-energy factor supply, that explain the fall in the price of energy services illustrated in Figure 8.

Appendix B Preferences and Demography

B.1 Proof of Theorem 1: Human Capital Accumulation

Assuming that the total stock of applied knowledge is initially insufficient for education expenditures to be strictly positive, that is $Q_t \leq \tilde{Q}_t$ initially, let us prove by complete induction that, whenever the education threshold is met, human capital accumulate. Let us denote by $\mathcal{P}_t$ the following property: $\mathcal{P}_t : h_t > h_{t-1}$.

Base case: assume that the total stock of knowledge is initially insufficient to meet the education threshold, and that there exist a period at which it is met.

Without loss of generality, let us rename the time index such that $t = 0$ is the period at which the education threshold is crossed (which corresponds to $t = T$ in theorem), that is $Q_0 > \tilde{Q}_0$ with $\tilde{Q}_0 = A_E^{-1}(\eta/\rho\tau)$. For $t < 0$, education expenditure are null and the level of human capital is constant at its minimum level, $\bar{h}$, as set by Eq. (4). Thus, $h_0 = \bar{h}$.

Besides, by definition of the education threshold, $e_0 > 0$ and then $h_1 > \bar{h}$ according to Eq. (4). As a result, $\mathcal{P}_1$ is true.

Induction: assume $\forall k \geq 0, k \leq t, \mathcal{P}_k$ is true.

The stock of total knowledge defined by Eq. (16) is strictly increasing because of learning-by-doing technical change defined by Eq. (21). Thus, we have

$$Q_t > Q_{t-1} > ... > Q_1 > Q_0.$$

Moreover, by induction, we know that $h_t > h_{t-1} > ... > h_1 > h_0$ and $h_0 = \bar{h}$. Hence, $\eta\bar{h}/\rho\tau h_t < \eta\bar{h}/\rho\tau h_{t-1} < ... < \eta\bar{h}/\rho\tau h_1 < \eta/\rho\tau$ and, by monotonicity and invertibility of $A_E(\cdot)$, we finally have

$$\tilde{Q}_t < \tilde{Q}_{t-1} < ... < \tilde{Q}_1 < \tilde{Q}_0.$$

Moreover, as recalled in the base case, we know that $Q_0 > \tilde{Q}_0$. Consequently,

$$\forall k \geq 0, k \leq t, Q_k > \tilde{Q}_k.$$

Then, provided the education threshold is met at each period $k \in [\![0; t]\!]$, the stock of total knowledge Q_t is cumulative, $A_E(\cdot)$ is increasing, and by induction we have

$$\begin{aligned} h_{t+1} - h_t &= A_E(Q_t)\frac{e_t}{w_t} - A_E(Q_{t-1})\frac{e_{t-1}}{w_{t-1}} \\ &= \frac{\rho\tau}{\eta - \rho}\left[A_E(Q_t)h_t - A_E(Q_{t-1})h_{t-1}\right] \\ &> \frac{\rho\tau A_E(Q_{t-1}}{\eta - \rho}\left[h_t - h_{t-1}\right] \\ &> 0. \end{aligned}$$

Hence, $\mathcal{P}_{t+1}$ is true.

B.2 Proof of Theorem 2: Quantity-Quality Trade-off

For an interior solution, combining the equilibrium allocations of time and income as given by Eqs. (31) and (32), we obtain the following fertility and education expenditures

$$b_t = \frac{(\eta - \rho) A_E(Q_t) h_t}{(1 + \chi + \eta)(A_E(Q_t) h_t \tau - h_0)}, \qquad e_t = \frac{\left[\rho \tau A_E(Q_t) h_t - \eta \bar{h}\right] w_t}{A_E(Q_t)(\eta - \rho)}.$$

Taking the partial derivatives with regards to η, ρ, w_t and h_t, one can check that $\partial b_t / \partial \eta > 0$, $\partial b_t / \partial \rho < 0$, $\partial e_t / \partial \eta < 0$, $\partial e_t / \partial \rho > 0$, $\partial b_t / \partial w_t = 0$, $\partial b_t / \partial h_t < 0$, $\partial e_t / \partial w_t > 0$, and $\partial e_t / \partial h_t > 0$, which completes the proof.

Appendix C Innovation and Production

C.1 Price of Capital Goods

The first order conditions from Problem FI yield the optimal demand schedules in each sector $k \in \mathcal{K}$ for capital goods, human capital and energy flows, according to

$$\frac{p^x_{k,i,t}}{p_{k,t}} = \alpha_k A_{k,t} H^{\beta_k}_{k,t} E^{\gamma_k}_{k,t} \left[\frac{q_{k,i,t}}{x_{k,i,t}}\right]^{1-\alpha_k}, \tag{44}$$

$$\frac{w_t}{p_{k,t}} = \beta_k \frac{Y_{k,t}}{H_{k,t}}, \tag{45}$$

$$\frac{\Psi_k}{p_{k,t}} = \gamma_k \frac{Y_{k,t}}{E_{k,t}}. \tag{46}$$

These conditions will be useful to derive further analytical results below. For now, we turn to Problem FI and distinguish the production regime according to the success of R&D, as exposed in Section 3.2.3. Whenever innovation is unsuccessful, the production is competitive and capital goods are thus priced at their marginal cost of production, that is $p^x_{k,i,t} = r_t + \delta$ (hereafter denoted $p^c_{k,i,t}$). Whenever innovation is successful, the production of capital goods is monopolistic. Substituting Eq. (44) within the operating profit, $\pi_{k,i,t}$, one can easily compute the monopolistic price charged by the successful innovator, $p^x_{k,i,t} = (r_t + \delta)/\alpha_k$ (hereafter denoted $p^m_{k,i,t}$).

C.2 Proof of Proposition 1: Sector-specific R&D Success Probability

Innovation decisions are driven by the one-period success monopoly profit arising from the improvement of a sector-specific machine line. Provided the monopolistic price of capital goods only depends on the related sector, one can write the success monopoly profit as

$$\pi^s_{k,i,t} = \bar{\pi}_{k,t} \left[\frac{1}{p^m_{k,t}}\right]^{\frac{\alpha_k}{1-\alpha_k}} q_k^{\kappa_{k,i,t}+1}, \tag{47}$$

where $\bar{\pi}_{k,t} = (1 - \alpha_k) \left[p_{k,t} \alpha_k A_{k,t} H^{\beta_k}_{k,t} E^{\gamma_k}_{k,t}\right]^{\frac{1}{1-\alpha_k}}$.

Substituting Eqs (47) and (24) into the free-entry condition of Problem R&D and isolating $\lambda_{k,i,t}$ immediately yields the result. The machine line index, i, can be dropped as the R&D success probability only depends on the corresponding sector.

C.3 Aggregation of Input Production

In this section, we derive the aggregation for final input sectors, $k \in \mathcal{K}$. First, remember that the production technology equation writes

$$Y_{k,t} = A_{k,t}\left[\int_0^1 q_{k,i,t}^{1-\alpha_k} x_{k,i,t}^{\alpha_k} di\right] H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}.$$

For each machine line $i \in [0,1]$, the optimal demand schedule is given by the first order condition of Eq. (44) with $p^x_{k,i,t}$ the price of the machine. As discussed in Subsection 3.2.3, this price depends on the status under which the machine is supplied. Whenever R&D was successful in the beginning of the period for the machine line, the machine is supplied under monopolistic competition at $p^m_{k,t} = (r_t+\delta)/\alpha_k$ and the corresponding optimal demand schedule equation then writes

$$x^m_{k,i,t} = \left[\frac{p_{k,t}\alpha_k^2 A_{k,t} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}}{r_t+\delta}\right]^{\frac{1}{1-\alpha_k}} q_{k,i,t}.$$

In the opposite case, the machine is supplied competitively and its price set at the marginal production cost, that is $p^c_{k,i,t} = r_t + \delta$. The corresponding optimal demand schedule equation then writes

$$x^c_{k,i,t} = \left[\frac{p_{k,t}\alpha_k A_{k,t} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}}{r_t+\delta}\right]^{\frac{1}{1-\alpha_k}} q_{k,i,t}.$$

As demonstrated in Subsection 3.3.3, R&D activity is uniformly distributed over machines lines with a probability of success $\lambda_{k,t}$. Thus, at the beginning of each period, a fraction $\lambda_{k,t}$ of firms experience a successful innovation, gain one additional rung of size q_k in their corresponding quality ladder, and are produced under monopolistic competition. For the remaining firms, the quality remains constant and the production is perfectly competitive. Discriminating firms accordingly and introducing the average quality index $Q_{k,t}$ (defined at the beginning of the period) yields

$$\begin{aligned} Y_{k,t} &= A_{k,t}\left[\int_0^{\lambda_{k,t}} q_{k,i,t}^{1-\alpha_k}(x^m_{k,i,t})^{\alpha_k} di + \int_{\lambda_{k,t}}^1 q_{k,i,t}^{1-\alpha_k}(x^c_{k,i,t})^{\alpha_k} di\right] H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}, \\ &= \left[A_{k,t} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}\right]^{\frac{1}{1-\alpha_k}} \left[\frac{p_{k,t}\alpha_k}{r_t+\delta}\right]^{\frac{\alpha_k}{1-\alpha_k}} \left[q_k \alpha_k^{\frac{\alpha_k}{1-\alpha_k}} \int_0^{\lambda_{k,t}} q_{k,i,t-1} di + \int_{\lambda_{k,t}}^1 q_{k,i,t-1} di\right], \\ &= \left[A_{k,t} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}\right]^{\frac{1}{1-\alpha_k}} \left[\frac{p_{k,t}\alpha_k}{r_t+\delta}\right]^{\frac{\alpha_k}{1-\alpha_k}} \left[\alpha_k^{\frac{\alpha_k}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})\right] Q_{k,t}. \end{aligned} \tag{48}$$

Turning now to the stock of capital ultimately dedicated to the raw capital demand in each final input sector, defined by $K_{k,t} = \int_0^1 x_{k,i,t} di$, one can distinguish between machines that are produced under monopolistic and perfect competition to write

$$K_{k,t} = \left[\frac{p_{k,t}\alpha_k A_{k,t} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_u}}{r_t+\delta}\right]^{\frac{1}{1-\alpha_k}} \left[\alpha^{\frac{1}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})\right] Q_{k,t-1}, \tag{49}$$

Inserting Eq (49) into Eq (48) and rearranging gives a final expression for the aggregate production function in the final good sector

$$Y_{k,t} = A_{k,t} Q_{k,t}^{1-\alpha_k} \frac{\alpha_k^{\frac{\alpha_k}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})}{\left[\alpha_k^{\frac{1}{1-\alpha_k}} q_k \lambda_{k,t} + (1-\lambda_{k,t})\right]^{\alpha_k}} K_{k,t}^{\alpha_k} H_{k,t}^{\beta_k} E_{k,t}^{\gamma_k}.$$

Appendix D Equilibrium Results

D.1 Proof of Proposition 2: Relative Input Price

Combining Eqs (44) and (46) allows to isolate resource flows as

$$E_{k,t} = \left[\frac{\alpha}{r_t+\delta}\right]^{\frac{\alpha}{\beta}} \left[\frac{\gamma \overline{Q}_{k,t} Q_{k,t}}{\Psi_{k,t}}\right]^{\frac{1-\alpha}{\beta}} \left[A_{k,t} p_{k,t}\right]^{\frac{1}{\beta}} H_{k,t}. \tag{50}$$

Substituting this expression and the equilibrium demand of the capital good (Eq. (44)) in Eq. (10) then yields the following expression for the equilibrium input production

$$Y_{k,t} = \left[\frac{\alpha}{r_t+\delta}\right]^{\frac{\alpha}{\beta}} \left[\overline{Q}_{k,t} Q_{k,t}\right]^{\frac{1-\alpha}{\beta}} \left[\frac{\gamma}{\Psi_{k,t}}\right]^{\frac{\gamma}{\beta}} A_{k,t}^{\frac{1}{\beta}} p_{k,t}^{\frac{1-\beta}{\beta}} H_{k,t}.$$

Equalizing the marginal product of labor across sectors, that is Eq. (45) taken for each sector, and using the previous relations then leads to

$$\frac{p_{r,t}}{p_{e,t}} = \left[\frac{A_{r,t}}{A_{e,t}}\right]^{-1} \left[\frac{\overline{Q}_{r,t} Q_{r,t}}{\overline{Q}_{e,t} Q_{e,t}}\right]^{-(1-\alpha)} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{\gamma}, \tag{51}$$

where $\overline{Q}_{k,t} = 1 + \lambda_{k,t}(\alpha^{\frac{\alpha}{1-\alpha}} q_k - 1)$ is a scaling factor for the corresponding sector-specific R&D knowledge. The proof then results in a direct interpretation of Eq. (51) abstracting from innovation decisions.

D.2 Proof of Proposition 3: Relative Factor Use

Substituting the equilibrium input production, Eq. (37) into Eq. (38), and combining the latter with (51), allows the isolate the equilibrium relative human capital allocation in input sectors as

$$\frac{H_{r,t}}{H_{e,t}} = \left[\frac{A_{r,t}}{A_{e,t}}\right]^{\sigma-1} \left[\frac{\overline{Q}_{r,t} Q_{r,t}}{\overline{Q}_{e,t} Q_{e,t}}\right]^{(1-\alpha)(\sigma-1)} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{-\gamma(\sigma-1)}. \tag{52}$$

A similar expression can be obtained for the equilibrium relative energy resource use by combining Eqs. (45) and (46) and substituting Eq. (52),

$$\frac{E_{r,t}}{E_{e,t}} = \left[\frac{A_{r,t}}{A_{e,t}}\right]^{\sigma-1} \left[\frac{\overline{Q}_{r,t} Q_{r,t}}{\overline{Q}_{e,t} Q_{e,t}}\right]^{(1-\alpha)(\sigma-1)} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{-\gamma(\sigma-1)-1}.$$

At last, substituting Eq. (44) into the sector-specific market clearing conditions for capital goods, Eq. (30), gives the following expression for the demand for physical capital,

$$K_{k,t} = \left[\frac{\alpha}{r_t+\delta}\right]^{\frac{\alpha+\beta}{\beta}} \overline{Q}_{k,t}^{\frac{\gamma}{\beta}} \widehat{Q}_{k,t} Q_{k,t}^{\frac{1-\alpha}{\beta}} \left[\frac{\gamma}{\Psi_{k,t}}\right]^{\frac{\gamma}{\beta}} \left[A_{k,t} p_{k,t}\right]^{\frac{1}{\beta}} H_{k,t},$$

with $\widehat{Q}_{k,t} = 1 + \lambda_{k,t}(\alpha^{\frac{1}{1-\alpha}} q_k - 1)$. Taking the ratio of this expression finally yields

$$\frac{K_{r,t}}{K_{e,t}} = \left[\frac{A_{r,t}}{A_{e,t}}\right]^{\sigma-1} \left[\frac{\widehat{Q}_{r,t}}{\widehat{Q}_{e,t}}\right] \left[\frac{\overline{Q}_{r,t}}{\overline{Q}_{e,t}}\right]^{(1-\alpha)(\sigma-1)-1} \left[\frac{Q_{r,t}}{Q_{e,t}}\right]^{(1-\alpha)(\sigma-1)} \left[\frac{\Psi_{r,t}}{\Psi_{e,t}}\right]^{-\gamma(\sigma-1)}.$$

Abstracting from innovation decisions, a direct assessment of these expressions finally yields the discussed property.

Appendix E Best-fit Calibration Process

In this section, we provide some technical information about our numerical simulation process and its calibration on historical data.

E.1 Simulation Process

We use the formal calculation software *Maple* (2017 version) to perform the numerical simulation of our model. As we cannot provide a closed form solution for the general version of our model, we use optimization commands (*Optimization* package) to solve for prices and quantities, accounting for market clearing and physical constraint conditions, as well as optimal (and closed-form) responses of the representative household. Our Monte-Carlo simulations are also performed on this software using the *Statistics* and *RandomTools* packages. For our numerical results, we performed a total of 10,000 runs on each scenario analyzed in Section 5. All the presented figures are realized with the *RStudio* software.

Provided that fertility, savings, and educational choices are endogenous in our model according to Problem HH (and thus depend on the clearing wage rate), we allow for two shimming periods (1660 and 1680) before starting the numerical simulations. These shimming periods ensure that household's decisions, population's allocation (between young and retired adults) and capital accumulation (resulting from previous savings) consistently adjust prior to the formal start of the simulation process. We preclude any knowledge accumulation during this shimming time, so knowledge stocks remain consistent with their normalization to unity in the first simulation period. It is worth mentioning that the current stock of capital only depends on savings from the previous period according to Eq. (5), therefore, the initial value chosen for the stock of capital does not matter for capital accumulation after these shimming periods.

E.2 Calibration Process

Due to the high dimensionality of our calibration problem and the complexity of the general equilibrium interactions in the full version of our model, we adopt a three-step calibration process:

- *Preferences calibration problem*: for given series of wage rate and total applied knowledge stock, the closed form solution of Problem HH allows us to obtain the partial equilibrium solution of the full model for population and human capital per capita. In a first calibration step, we thus optimize for household preferences and the initial population stock to fit these two time series, that is, we set $\{\chi, \rho, \sigma, \tau, A_E, N_0\}$ to minimize the squared error between the simulated model and historical data.[33]

- *Deterministic calibration problem*: for a given series of GPT arrivals (determined according to historical data), solving numerically for Problem GE allows us to obtain a deterministic solution of our model. In a second calibration step, we thus optimize for all parameters except GPT related parameters to fit our historical times series of interest. That is, we set $\{Y_0, \Omega, \omega_H, \omega_G, \phi, q_c, q_d, \bar{\lambda}, \overline{\Psi}_r, \overline{\Psi}_e, \psi_{\mathcal{R},r}, \psi_{\mathcal{R},e}, \psi_Q\}$ to minimize various squared error metrics between the simulated model and historical data on GDP, population, energy consumption flows and human capital per

[33] We use a direct method algorithm from the *DirectSearch* package to cover the whole compact calibration set. We give 10 times greater weight to population than to human capital per capita (squared-error) scores, and use a Hodrick-Prescott (HP) filter to smooth historical time series so as to downplay the extreme values of the Great Depression and World War (I and II). N_0 denotes the population stock that will be applied to the shimming periods. To account for this slight approximation (the population however being initially low) we perform the *preference calibration problem* in the 1660-2000 period to account for the two shimming periods.

capita.[34] For each assessed design, we also optimize for A_E to minimize the squared error in regards to population and human capital per capita (with a 10 times lower weight for the second) to ease the coupling between the *preferences* and *deterministic calibration problems*.

– *Stochastic GPT calibration problem*: for a given calibration design (chosen as the best candidate from the *deterministic calibration problem*), repeatedly solving for Problem GE allows us to obtain a Monte-Carlo solution for our full model. In practice, due to computational constraints, we performed 100 runs for each calibration design assessed in this step. This number is gauged to be a good approximation for Monte-Carlo convergence. We tested the designs that score best on a higher number of steps (1,000 for the calibration process, 10,000 for the numerical results). In this last calibration step, we thus optimize for GTP related parameters only to fit the historical sequence of GPT arrival, that is, we set $\{\mu_0, \bar{G}, \zeta, \xi\}$ to minimize the squared errors between the simulated and historical GDP frequency.[35] In this calibration step, given that we assess the relationship between total applied knowledge accumulation, Q_t, and GPT's frequency and diffusion, we only performed this step for the British calibration and maintain the obtained parameters for the Western European and Eastern Asian cases. As illustrated in additional Figure 9, we observe that the frequency of GPTs in the *baseline* calibration of Western Europe closely follows the one obtained for Great Britain, while it is lower for Eastern Asia, as one would logically expect.

If necessary (i.e., whenever a significant difference is observed), these three steps are cyclically repeated until the calibration converged within an increasing series of compact sets for LHS procedures (at each repeated step, we removed ranges of parameters that yielded the poorest results based on the best 2 to 3 runs on each dimension of interest).

Appendix F Baseline Calibration for Great Britain

Table 5 and 6 respectively present the initial variables and parameters (rounded to the nearest 10^{-3}) obtained for the baseline calibration of the model to the British data. If no index $k \in \{r, e\}$ is provided (e.g., A), the initial condition or parameter concerns both sectors.

Table 5 Initial values of variables for the *baseline* calibration of the model on British data.

Variable	A_0	G_0	$\bar{h}$	K_0	N_0	Q_0	$\mathcal{R}_{r,0}$	$\mathcal{R}_{e,0}$
Initial value	1	1	1	0.002	0.094	1	1.073	463.097

[34] We use a Latin Hypercube Sampling (LHS) with a *maximin* criteria to test different sets with up to 3200 samples from a compact calibration set. The sampling is performed using the *lhs* package used on the *RStudio* software. We computed various squared-errors metrics between the simulated model and historical data, that are smoothed with a HP-filter to downplay the extreme values of the Great Depression and World War (I and II). More precisely, each variable in $\{E_r, E_e, E_r/E_e, \mathcal{R}_r, \mathcal{R}_e, Y, h, P, Y/P\}$ is mapped to a constructed and normalized historical time series, from which we compute a squared-error and weighted squared-error (allowing for a 10 times greater weight for periods prior 1900). For the British case, we then pick designs that perform well favoring (i) the weighted Y score (fit on GDP during the economic take-off), and (ii) the sum of squared-error differences for the score of variables in $\{E_r/E_e, \mathcal{R}_r, \mathcal{R}_e, Y, h, P, Y/P\}$ (fit on the main time-series of interest on the whole simulation horizon). For both Western Europe and Eastern Asia calibrations, we pick designs that perform well according to the weighed sum of squared-error differences for the score of variables in $\{E_e, E_r, Y, h, P, Y/P\}$. In the Western European case, we emphasize the pre-WWII period, provided the industrial take-off and transition towards fossil fuels occurred in the XIX$^{\text{th}}$ century. For this reason we ascribe 10 times more weight to the squared-error differences prior 1940. In the Eastern Asian case, we emphasize the post-1900 period provided the industrial take-off and transition towards fossil energy occurred in the XX$^{\text{th}}$ century. This is why we ascribe 10 times more weight to the squared-error differences after 1900.

[35] We use a LHS procedure with a *maximin* criteria to test different sets with up to 1600 samples from a compact calibration set. The sampling is performed using the *lhs* package used on the *RStudio* software.

Table 6 Parameters for the *baseline* calibration of the model on British data.

Parameter	α	β	γ	ϑ_k	Y_0	Ω	ω_H	ω_G	ϕ
Value	1/3	1/2	1/6	1	3.3	0.012	0.027	0.445	0.183
Parameter	q	$\bar{\lambda}$	$\overline{\Psi}_r$	$\overline{\Psi}_e$	$\psi_{\mathcal{R},r}$	$\psi_{\mathcal{R},e}$	ψ_Q	η	χ
Value	1.26	0.377	0.016	0.106	-4.506	-7.296	-7.993	0.202	0.3
Parameter	ρ	τ	A_E	$\bar{h}$	μ_0	$\bar{G}$	ζ	ξ	
Value	0.048	0.102	51.74	1	0.1	1.69	0.179	0.244	

Appendix G Baseline Calibration for Western Europe

Table 7 and 8 respectively present the initial variables and parameters (rounded to the nearest 10^{-3}) obtained for the baseline calibration of the model for Western Europe. If no index $k \in \{r, e\}$ is provided (e.g., A), the initial condition or parameter concerns both sectors.

Table 7 Initial values of variables for the *baseline WE* calibration

Variable	A_0	G_0	$\bar{h}$	K_0	N_0	Q_0	$\mathcal{R}_{r,0}$	$\mathcal{R}_{e,0}$
Initial value	1	1	1	0.04	0.018	1	5.76	227.19

Table 8 Parameters for the *baseline WE* calibration

Parameter	α	β	γ	ϑ_k	Y_0	Ω	ω_H	ω_G	ϕ
Value	1/3	1/2	1/6	1	3.145	0.014	0.059	0.492	0.179
Parameter	q_r	q_e	$\bar{\lambda}$	$\overline{\Psi}_r$	$\overline{\Psi}_e$	$\psi_{\mathcal{R},r}$	$\psi_{\mathcal{R},e}$	ψ_Q	η
Value	1.361	1.371	0.686	0.03	0.192	-8	-4.209	-4.14	0.479
Parameter	χ	ρ	τ	A_E	μ_0	$\bar{G}$	ζ	ξ	
Value	0.37	0.037	0.224	73.696	0.1	1.69	0.179	0.244	

Appendix H Baseline Calibration for Eastern Asia

Table 9 and 10 respectively present the initial variables and parameters (rounded to the nearest 10^{-3}) obtained for the baseline calibration of the model to the Eastern Asia. If no index $k \in \{r, e\}$ is provided (e.g., A), the initial condition or parameter concerns both sectors.

Table 9 Initial values of variables for the *baseline EA* calibration

Variable	A_0	G_0	$\bar{h}$	K_0	N_0	Q_0	$\mathcal{R}_{r,0}$	$\mathcal{R}_{e,0}$
Initial value	1	1	1	0.005	0.022	1	20.43	72.57

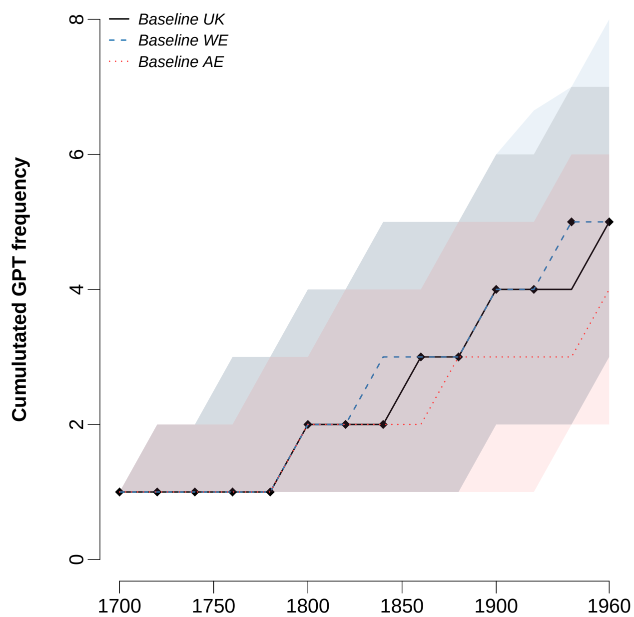
Table 10 Parameters for the *baseline EA* calibration

Parameter	α	β	γ	ϑ_k	Y_0	Ω	ω_H	ω_G	ϕ
Value	1/3	1/2	1/6	1	2.294	0.001	0.044	0.335	0.184
Parameter	q_r	q_e	$\bar{\lambda}$	$\overline{\Psi}_r$	$\overline{\Psi}_e$	$\psi_{\mathcal{R},r}$	$\psi_{\mathcal{R},e}$	ψ_Q	η
Value	1.179	1.55	0.789	0.01	0.222	-5.221	-3.182	-5.278	0.431
Parameter	χ	ρ	τ	A_E	μ_0	$\bar{G}$	ζ	ξ	
Value	0.358	0.052	0.179	58.626	0.1	1.69	0.179	0.244	

Appendix I GPT Frequency in Baseline Calibrations

GPTs are calibrated on the British case. The values obtained for the parameters $\{\mu_0, \bar{G}, \xi, \zeta\}$ are assumed to reflect the relationship between total applied knowledge, Q_t, and the arrival and diffusion time of GPTs. These values are consequently maintained for both Western Europe and Eastern Asia. In Figure 9, one can observe that GPT frequency closely followed the British case for Western Europe, but are consistently lower for Eastern Asia, as one would logically expect.

Fig. 9 GPT frequency in the *baseline* calibration (10,000 simulations) for Great Britain *(black)*, Western Europe *(blue)* and Eastern Asia *(red)*, 1700-1960 *(solid, dashed and dotted lines are medians, color shades are 90% probability intervals, diamonds are historical data)*. *Data source*: Lipsey *et al.* (2005, p. 132)



References

Acemoglu, Daron. 2002. Directed Technical Change. *Review of Economic Studies*, **69**(4), 781–809.

Acemoglu, Daron. 2009. *Introduction to Modern Economic Growth*. Princeton, NJ: Princeton University Press.

Acemoglu, Daron, Aghion, Philippe, Bursztyn, Leonardo, & Hemous, David. 2012. The Environment and Directed Technical Change. *American Economic Review*, **102**(1), 131–166.

Aghion, Philippe, & Howitt, Peter. 1992. A Model of Growth Through Creative Destruction. *Econometrica*, **60**(2), 323–351.

Aghion, Philippe, & Howitt, Peter. 1998. *Endogenous Growth Theory*. Cambridge, MA: MIT Press.

Aghion, Philippe, & Howitt, Peter. 2009. *The Economics of Growth*. Cambridge, MA: MIT Press.

Allen, R. C. 2011. Why the industrial revolution was British: commerce, induced invention, and the scientific revolution. *Economic History Review*, **64**(2), 357–384.

Allen, Robert C. 2009. *The British Industrial Revolution in Global Perspective*. Cambridge, UK: Cambridge University Press.

Allen, Robert C., Bassino, Jean-Pascal, Ma, Debin, Moll-Murata, Christine, & van Zanden, Jan Luiten. 2011. Wages, prices, and living standards in China, 1738-1925: in comparison with Europe, Japan, and India. *Economic History Review*, **64**(S1), 8–38.

Angrist, Joshua, Lavy, Victor, & Schlosser, Analia. 2010. Multiple Experiments for the Causal Link between the Quantity and Quality of Children. *Journal of Labor Economics*, **28**(4), 773–824.

Ayres, Robert U., & Warr, Benjamin. 2009. *The Economic Growth Engine: How Energy and Work Drive Material Prosperity*. Cheltenham, UK: Edward Elgar Publishing.

Ayres, Robert U., van den Bergh, Jeroen C.J.M., Lindenberger, Dietmar, & Warr, Benjamin. 2013. The underestimated contribution of energy to economic growth. *Structural Change and Economic Dynamics*, **27**, 79–88.

Becker, Gary Stanley. 1960. An Economic Analysis of Fertility. *In:* Becker, Gary Stanley (ed), *Demographic and Economic Change in Developed Countries*. Princeton, NJ: Princeton University Press.

Becker, Gary Stanley. 1981. *A Treatise on the Family*. Cambridge, MA: Harvard University Press.

Becker, Sascha O., Cinnirella, Francesco, & Woessmann, Ludger. 2010. The trade-off between fertility and education: evidence from before the demographic transition. *Journal of Economic Growth*, **15**, 177–204.

Berg, Maxine. 1994. *The Age of Manufactures, 1700–1820: Industry, Innovation, and Work in Britain*. London, UK: Routledge.

Black, Sandra E., Devereux, Paul J., & Salvanes, Kjell. G. 2005. The More the Merrier ? The Effect of Family Size and Birth Order on Children's Education. *Quarterly Journal of Economics*, **120**(2), 669–700.

Bolt, Jutta, Inklaar, Robert, de Jong, Herman, & van Zanden, Jan Luiten. 2018. *Maddison Project Database 2018*. https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2018.

Boucekkine, Raouf, de la Croix, David, & Peeters, Dominique. 2007. Early literacy achievements, population density, and the transition to modern growth. *Journal of the European Economic Association*, **5**(1), 183–226.

British Petroleum. 2015. *Statistical Review of World Energy 2015*. Tech. rept.

Broadberry, Stephen, Campbell, Bruce M. S., Klein, Alexander, Overton, Mark, & Leeuwen, Bas van. 2015. *British Economic Growth, 1270–1870*. Cambridge, UK: Cambridge University Press.

Broadberry, Stephen, Guan, Hanhui, & Li, David Daokui. 2018. China, Europe, and the Great Divergence: a study in historical national accounting, 980–1850. *Journal of Economic History*, **78**(4), 1–46.

Cáceres-Delphiano, Julio. 2006. The Impacts of Family Size on Investment in Child Quality. *Journal of Human Resources*, **41**(4), 738–754.

Cervellati, Matteo, & Sunde, Uwe. 2005. Human Capital Formation, Life Expectancy, and the Process of Development. *American Economic Review*, **95**(5), 1653–1672.

Clark, G., & Jacks, D. 2007. Coal and the Industrial Revolution, 1700-1869. *European Review of Economic History*, **11**(1), 39–72.

Clark, Gregory, & Cummins, Neil. 2016. The Child Quality-Quantity Tradeoff, England, 1780-1880: A Fundamental Component of the Economic Theory of Growth is Missing. *CEPR Discussion Papers*, **No. DP1123**.

Court, V., Jouvet, P.A., & Lantz, F. 2018. Long-term endogenous economic growth and energy transitions. *Energy Journal*, **39**(1), 29–57.

Crosby, Alfred W. 2007. *Children of the Sun: A History of Humanity's Unappeasable Appetite for Energy*. New York, NY: W.W. Norton & Company.

de La Croix, David, & Michel, Philippe. 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge, UK: Cambridge University Press.

de Pleijt, A.M., Nuvolari, A., & Weisdorf, J.L. 2019. Human capital formation during the first Industrial Revolution: evidence from the use of steam engines. *Journal of the European Economic Association*, **17**(in press).

de Vries, Jan. 1994. The Industrial Revolution and the Industrious Revolution. *Journal of Economic History*, **54**(2), 249–270.

Denison, Edward Fulton. 1979. *Accounting for slower economic growth : the United States in the 1970s*. Washington, DC: The Brookings Institution.

Desmet, Klaus, & Parente, Stephen L. 2012. The evolution of markets and the revolution of industry: a unified theory of growth. *Journal of Economic Growth*, **17**(3), 205–234.

Doepke, Matthias. 2004. Accounting for Fertility Decline During the Transition to Growth. *Journal of Economic Growth*, **9**, 347–383.

Fernihough, Alan, & O'Rourke, Kevin Hjortshøj. 2014. Coal and the European Industrial Revolution. *NBER Working Paper Series*, **19802**, 1–60.

Fizaine, Florian, & Court, Victor. 2016. Energy expenditure, economic growth, and the minimum EROI of society. *Energy Policy*, **95**, 172–186.

Fouquet, Roger. 2011. Divergences in Long Run Trends in the Prices of Energy and Energy Services. *Review of Environmental Economics and Policy*, **5**(2), 196–218.

Fouquet, Roger. 2014. Long run demand for energy services: income and price elasticities over 200 years. *Review of Environmental Economics and Policy*, **8**(2), 186–207.

Fouquet, Roger, & Broadberry, Stephen. 2015. Seven Centuries of European Economic Growth and Decline. *Journal of Economic Perspectives*, **29**(4), 227–244.

Fröling, Maria. 2011. Energy use, population and growth, 1800–1970. *Journal of Population Economics*, **24**(3), 1133–1163.

Galindev, Ragchaasuren. 2011. Leisure goods, education attainment and fertility choice. *Journal of Economic Growth*, **16**(2), 157–181.

Galor, Oded. 2005. From Stagnation to Growth: Unified Growth Theory. *Chap. 4 of:* Aghion, P., & Durlauf, S.N. (eds), *Handbook of Economic Growth, Vol. 1A*, vol. 1A. Amsterdam, NL: Elsevier B.V.

Galor, Oded, & Michalopoulos, Stelios. 2012. Evolution and the growth process: Natural selection of entrepreneurial traits. *Journal of Economic Theory*, **147**, 759–780.

Galor, Oded, & Moav, Omer. 2002. Natural Selection and the Origin of Economic Growth. *Quarterly Journal of Economics*, **117**(4), 1133–1191.

Galor, Oded, & Weil, David N. 2000. Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond. *American Economic Review*, **90**(4), 806–828.

Gars, Johan, & Olovsson, Conny. 2019. Fuel for Economic Growth? *Journal of Economic Theory*, **184**, 104–941.

Gentvilaite, Ruta, Kander, Astrid, & Warde, Paul. 2015. The Role of Energy Quality in Shaping Long-Term Energy Intensity in Europe. *Energies*, **8**(1), 133–153.

Goldstone, Jack A. 2002. Efflorescences and Economic Growth in World History: Rethinking the "Rise of the West" and the Industrial Revolution. *Journal of World History*, **13**(2), 323–389.

Goldstone, Jack A. 2009. *Why Europe? The Rise of the West in World History, 1500-1850*. Boston, MA: McGraw-Hill Higher Education.

Hazan, Moshe, & Zoabi, Hosny. 2006. Does longevity cause growth? A theoretical critique. *Journal of Economic Growth*, **11**, 363–376.

IIASA. 2012. *Global Energy Assessment - Toward a Sustainable Future*. Tech. rept. International Institute for Applied Systems Analysis, Schloss Laxenburg, AT.

Jacob, Margaret C. 2014. *The First Knowledge Economy: Human Capital and the European Economy, 1750-1850*. Cambridge , UK: Cambridge University Press.

Jones, Charles I. 1995. R&D-Based Models of Economic Growth. *Journal of Political Economy*, **103**(4), 759–784.

Jones, Charles I. 2001. Was an Industrial Revolution Inevitable? Economic Growth Over the Very Long Run. *Advances in Macroeconomics*, **1**(2), 1–43.

Kander, Astrid, & Stern, David I. 2014. Economic growth and the transition from traditional to modern energy in Sweden. *Energy Economics*, **46**(11), 56–65.

Kander, Astrid, Malanima, Paolo, & Warde, Paul. 2013. *Power to the People: Energy in Europe Over the Last Five Centuries*. Princeton, NJ: Princeton University Press.

Kilian, Lutz. 2008. The Economic Effects of Energy Price Shocks. *Journal of Economic Literature*, **46**(4), 871–909.

Klemp, Marc, & Weisdorf, Jacob. 2018. Fecundity, Fertility and the Formation of Human Capital. *Economic Journal*, 6.

Kortum, Samuel. 1993. Equilibrium R&D and the Patent–R&D Ratio: U.S. Evidence. *American Economic Review*, **83**(2), 450–457.

Kümmel, Reiner. 2011. *The Second Law of Economics: Energy, Entropy, and the Origins of Wealth*. New York, NY: Springer.

Kümmel, Reiner, & Lindenberger, Dietmar. 2014. How energy conversion drives economic growth far from the equilibrium of neoclassical economics. *New Journal of Physics*, **16**(12), 125008.

Lagerlöf, Nils-Petter. 2003. Gender Equality and Long-Run Growth. *Journal of Economic Growth*, **8**, 403–426.

Lee, Jong-Wha, & Lee, Hanol. 2016. Human capital in the long run. *Journal of Development Economics*, **122**(9), 147–169.

Lemoine, Derek. 2018. Innovation-Led Transitions in Energy Supply. *NBER Working Paper Series*, **23420**.

Li, Hongbin, Zhang, Junsen, & Zhu, Yi. 2008. The Quantity-Quality Trade-off of Children in a Developing Country: Identification Using Chinese Twins. *Demography*, **45**(1), 223–243.

Lipsey, Richard G., Carlaw, Kenneth., & Bekar, Clifford. 2005. *Economic Transformations: General Purpose Technologies and Long-Term Economic Growth*. Oxford, UK: Oxford University Press.

Maddison, Angus. 2007. *Chinese Economic Performance in the Long Run – Second edition, revised, and updated: 960-2030 AD*. Paris, FR: OECD Development Centre Studies.

Malanima, P. 2011. The long decline of a leading economy: GDP in Central and Northern Italy, 1300-1913. *European Review of Economic History*, **15**(2), 169–219.

Malanima, Paolo. 2009. *Pre-modern European Economy : One Thousand Years (10th-19th Centuries)*. Leiden, NL: Brill.

Malanima, Paolo. 2016. Energy consumption in England and Italy, 1560-1913. Two pathways toward energy transition. *Economic History Review*, **69**(1), 78–103.

Malm, Andreas. 2013. The origins of fossil capital: From water to steam in the British cotton industry. *Historical Materialism*, **21**(1), 15–68.

Mokyr, Joel. 1990. *The Lever of Riches: Technological Creativity and Economic Progress*. New York, NY: Oxford University Press.

Mokyr, Joel. 2011. *The Enlightened Economy: Britain and the Industrial Revolution, 1700-1850*. London, UK: Penguin Books.

Mokyr, Joel. 2017. *A Culture of Growth: The Origins of the Modern Economy*. Princeton, NJ: Princeton University Press.

Morris, Max D., & Mitchell, Toby J. 1995. Exploratory designs for computational experiments. *Journal of Statistical Planning and Inference*, **43**(3), 381–402.

Otojanov, Ravshonbek. 2018. Energy Transitions, Directed Technical Change and the British Industrial Revolution. *Centre for Globalisation Research Working Paper Series*, **91**.

O'Rourke, Kevin Hjortshøj, Rahman, Ahmed S., & Taylor, Alan M. 2013. Luddites, the industrial revolution, and the demographic transition. *Journal of Economic Growth*, **18**(4), 373–409.

Pomeranz, Kenneth. 2000. *The Great Divergence: China, Europe, and the Making of the Modern World Economy*. Princeton, NJ: Princeton University Press.

Schaefer, Andreas, Schiess, Daniel, & Wehrli, Roger. 2014. Long-term growth driven by a sequence of general purpose technologies. *Economic Modelling*, **37**(2), 23–31.

Squicciarini, Mara P., & Voigtländer, Nico. 2015. Human Capital and Industrialization: Evidence from the Age of Enlightenment. *Quarterly Journal of Economics*, **130**(4), 1825–1883.

Stern, David I., Pezzey, John C. V., & Lu, Yingying. 2019. Directed Technical Change and the British Industrial Revolution. *USAEE Working Paper*, **19-409**.

Stokey, Nancy L. 1995. R and D, and Economic Growth. *Review of Economic Studies*, **62**(3), 469–489.

Strulik, Holger, & Weisdorf, Jacob. 2008. Population, food, and knowledge: a simple unified growth theory. *Journal of Economic Growth*, **13**(3), 195–216.

Strulik, Holger, Prettner, Klaus, & Prskawetz, Alexia. 2013. The past and future of knowledge-based growth. *Journal of Economic Growth*, **18**(4), 411–437.

Warde, Paul. 2007. *Energy Consumption in England & Wales, 1560–2000*. Tech. rept. Consiglio Nazionale delle Ricerche, Istituto di Studi sulle Società del Mediterraneo.

World Bank. 2018. *World Bank Indicators, population, total*.

Wrigley, E. A, Davies, R. S., Oeppen, J. E., & Schofield, R. S. 1997. *English Population History from Family Reconstitution, 1580-1837*. Cambridge, UK: Cambridge University Press.

Wrigley, Edward Anthony. 2016. *The Path to Sustained Growth: England's Transition from an Organic Economy to an Industrial Revolution*. Cambridge, UK: Cambridge University Press.

Yakita, Akira. 2010. Human capital accumulation, fertility and economic development. *Journal of Economics*, **99**(2), 97–116.

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